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Utilitarian Approaches for Multi-Metric Optimization in VLSI Circuit Design and Spatial Clustering

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by

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DEDICATION

To my wonderful parents,

with all my love and respect.

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NOTE TO READER

The original of this document contains color that is necessary for understanding the data. The original dissertation is on file with the University of South Florida library in Tampa, Florida.

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UTILITARIAN APPROACHES FOR MULTI-METRIC OPTIMIZATION IN VLSI CIRCUIT DESIGN AND SPATIAL CLUSTERING

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ABSTRACT

In the field of VLSI circuit optimization, the scaling of semiconductor devices has led to the miniaturization of the feature sizes resulting in a significant increase in the integration density and size of the circuits. At the nanometer level, due to the effects of manufacturing process variations, the design optimization process has transitioned from the deterministic domain to the stochastic domain, and the inter-relationships among the specification parameters like delay, power, reliability, noise and area have become more intricate. New methods are required to examine these metrics in a unified manner, thus necessitating the need for multi-metric optimization. The optimization algorithms need to be accurate and efficient enough to handle large circuits. As the size of an optimization problem increases significantly, the ability to cluster the design metrics or the parameters of the problem for computational efficiency as well as better analysis of possible trade-offs becomes critical. In this dissertation research, several utilitarian methods are investigated for variation aware multi-metric optimization in VLSI circuit design and spatial pattern clustering.

A novel algorithm based on the concepts of utility theory and risk minimization is developed for variation aware multi-metric optimization of delay, power and crosstalk noise, through gate sizing. The algorithm can model device and interconnect variations independent of the underlying distributions and works by identifying a deterministic linear equivalent model from a fundamentally stochastic optimization problem. Furthermore, a multi-metric gate sizing optimization framework is developed that is independent of the optimization methodology, and can be implemented using any mathematical programming approach. It is generalized and reconfigurable such that the metrics can be selected, removed, or prioritized for relative importance depending upon the design requirements.

In multi-objective optimization, the existence of multiple conflicting objectives makes the clustering problem challenging. Since game theory provides a natural framework for examining conflicting situations, a game theoretic algorithm for multi-objective clustering is introduced in this dissertation research. The problem of multi-metric clustering is formulated as a normal form multi-step game and solved using Nash equilibrium theory. This algorithm has useful applications in several engineering and multi-disciplinary domains which is illustrated by its mapping to the problem of robot team formation in the field in multi-emergency search and rescue.

The various algorithms developed in this dissertation achieve significantly better optimization and run times as compared to other methods, ensure high utility levels, are deterministic in nature and hence can be applied to very large designs. The algorithms have been rigorously tested on the appropriate benchmarks and data sets to establish their efficacy as feasible solution methods. Various quantitative sensitivity analysis have been performed to identify the inter-relationships between the various design parameters.

CHAPTER 1

INTRODUCTION

The advances in science and technology impact the realm of engineering. The most important facet of the technology evolution is that it facilitates the development of improved products, and helps in applying the knowledge and intelligence gained from one discipline to advance other disciplines. The important objectives in developing these products are, incorporation of enhanced feature sets, improvement in performance, and miniaturization. One way to achieve these objectives is to scale down the dimensions of various constituent elements or components of these products so that more components can be integrated on it. Improvements in the fabrication technologies aid in achieving these goals. However, the transition from one technology level to another is not rudimentary, and it uncovers new concerns. In the context of very large scale integrated computer aided design (VLSI-CAD), specifically circuit optimization, these concerns can be explained as follows.

With the aggressive scaling of semiconductor devices to the nano-meter level, the integration density of the circuits increases. According to the International Technology Roadmap for Semiconductors (ITRS) [2], the feature sizes for the devices and interconnects will continue to scale down at the rate of 0.7x per generation. This reduction in sizes affect the circuit optimization process in several ways. First, as the wiring density and consequently the aspect ratios in the metal lines increase, the cross-coupling capacitance between the neighboring interconnects grows. This may result in an increase in the interconnect crosstalk noise on a wire, due to the charge injected in it during the switching in the neighboring nets. In the deep sub-nanometer designs, such coupling capacitance effects between the adjacent nets can cause functionality failures causing reliability issues [3]. The noise due to cross-coupling capacitance is a dominant component among the noise sources, and hence is an essential consideration during the circuit optimization process. Second, the demand for power sensitive devices has grown significantly in recent years. This is attributed to the remarkable growth of personal computing and mobile devices such as laptop computers, cellular phones, music players and other portable

devices that are predominantly battery driven. These devices demand high-speed computational functionalities with low power consumption. However, as the integration density of transistors in a die and the frequency of operations increase, the power consumption in a die increases with each generation. To maintain low power dissipation, supply voltage is scaled down. However, the scaling of supply voltage is limited by the high-performance requirements. In order to maintain the performance, the transistor threshold voltage should be scaled down to achieve low switching energy per device. Scaling of threshold voltage significantly increases the sub-threshold leakage current [4], resulting in high leakage power dissipation during standby. Thus, at the sub-nanometer level, power minimization is an important metric in the circuit optimization process along with the performance metric. Hence, with the scaling of technology, new paradigms that impact the performance and reliability of the designs become an integral part of the design and optimization process.

The inter-relationships between these optimization metrics have become more intricate in the nano-meter regime. Optimization of one metric alone may result in a performance shift from one metric to another, thereby introducing sub-optimality in the values of other metrics. As a simple example, if some circuit optimization technique is employed with an objective of only power minimization, the resulting circuit configuration may potentially have high interconnect crosstalk noise, and hence low signal reliability. Alternatively, if the optimization is performed with the objective of crosstalk noise minimization, the resulting design may not be low power dissipating, thereby affecting the performance of the device. Addressing these aspects of optimization are important considerations in the next generation circuit optimization.

As the process technology is scaled down, the limitations due to manufacturing processes and environmental noise, make the physical realization of devices and interconnects unpredictable during the front-end design. During the fabrication of semiconductor devices, the existence of non-uniform conditions at the deposition and diffusion stages, or due to the limited resolution of the photolithographic process, the parameters like oxide thickness, effective gate length of individual transistors and interconnect widths may not follow the specifications. These variations may result in dramatic changes in the device performance characteristics, as well as the reliability of the designs. As a result, the design and optimization problem has transitioned from the deterministic domain to the probabilistic domain [5]. Also, these process tolerances do not scale proportionally, thereby increasing the relative impact of process variation on the design process with each new technology node.

This transition of optimization process to the stochastic domain affects the circuit optimization process. Since, the stochastic optimization techniques are inherently slower than their deterministic equivalents, the optimization process is adversely affected. In recent years, the state of the art research in VLSI design automation has addressed this issue. Several circuit optimization methods have been developed with an objective of centering the designs specifications such that majority of the fabricated circuits follow design and performance specifications. Many of these methods are based on the assumptions that the variation sources of the components follow specific distributions, such as Gaussian distribution, identified during the preliminary analysis [6, 7]. However, recent research refute such assumptions [8, 9]. Additionally, more sources of process variation are becoming predominant as the level of miniaturization is increasing, which is a principal concern in the semiconductor industry.

As a result of technology scaling, more components are integrated on the design area. Considering a simple example, the recent Intel Itanium®processor, code named 'Tukwila', released in 2008 is a two billion transistor chip [10] manufactured with 65nm technology. The total area of the chip is 699mm^2 as compared to the billion transistor Itanium 'Montecito' chip with a design area of 580mm^2. Although the number of transistors have doubled, the chip area has increase only by 20%. Due to the increase in the number of components, and consequently the problem size, the optimization process becomes significantly slow. In a general optimization problem, the size of a problem can be reduced by partitioning it into several smaller clusters, and performing optimization in each cluster separately. However, the clustering problem is not elementary, and any technique developed specifically for clustering of data objects in one knowledge discipline may not be directly applicable for clustering in other disciplines.

In spatial pattern clustering, several techniques have been developed for various applications in a wide variety of scientific disciplines such as biology, computer vision and pattern recognition, and communications and computer networks [11,12]. These techniques are largely application specific and perform single metric optimization. Hence, they may not be applicable to the applications like VLSI design partitioning, rescue robots deployment, ad-hoc networks establishment, and multi-emergency resource management etc. Often, multiple competitive metrics are required to be targeted for optimization in these engineering domains. To understand this problem, we can consider a hypothetical multi-emergency environment where an ad-hoc network of nodes (rescue personnel, resources etc.) performing the rescue operations at different emergency locations is to be established over a wireless

link. Even though each node may have identical capabilities, due to the battery power constraints, a subset of nodes are required to be identified that would be responsible for inter- and intra-cluster communication. An optimal clustering mechanism must ensure that the nodes, as well as the complete clusters do not drop out of the network. A clustering performed on the basis of one metric, say *cluster compactness* for low power dissipation in intra-cluster communication, may result in a situation where some clusters are too large and some are too small. The non-uniform power distribution among the clusters in this case may result in a situation where the battery of the nodes in smaller clusters may soon get exhausted, and the nodes drop out of the system. This would result in loss of communication from the emergency locations these nodes were servicing.

The exponential nature of such clustering problems qualifies the application of heuristics based optimization methodologies. However, any heuristic approach may not be adequate for spatial clustering in this domain due to some inherent characteristics of these problems. First, the optimization metrics here are often competitive in nature, and hence can not be optimized using the classical heuristics based optimization methods that perform a single metric optimization, such as genetic algorithms, simulated annealing etc. The clustering problem described above, represent one such class of problems. The two objectives, cluster compactness and uniform power distribution are conflicting in nature and need to be optimized simultaneously. Second, in several applications of this type, each objective to be optimized during the clustering process is critical. In terms of the clustering performance, this translates to a situation where the success of a clustering methodology is ascertained by the mutual satisfaction of the optimizations corresponding to each objective in the problem. Formally, this metric of success is termed as the *social fairness* [13] of the system. A concept widely studied and used in the field of economics, social fairness of a system corresponds to a situation where each individual (or metric) in the system is satisfied with respect to every other individual in the system, and the overall goals are achieved. In this example, the social fairness of the partitioning mechanism for the ad-hoc network clustering problem is maximized if both the objectives, compactness and uniform power distribution are satisfied (optimized) with respect to each other. The mutual satisfaction ensures that all the metrics are considered with the same priority level, and at an equilibrium solution point, any improvement in one metric can only be achieved by worsening the optimization of other metrics.

The speed of technology evolution decides the lifetime of the products. The lifespan of the products is shrinking due to rapid improvement in the manufacturing technology. This entails the designers

to identify and develop generalized optimization methods capable of incorporating the design objectives of the future generation products, and are applicable to multiple disciplines with relative ease. The objectives may include examination of additional metrics during optimization, and investigation and incorporation of the effect of randomness at several levels. The optimization frameworks capable of addressing these issues effectively would be beneficial for the community.

1.1 Motivation

The issues discussed above give a strong intuition about the problems that will be prevalent in the next generation computer engineering research. More precisely, in the VLSI-CAD, the technology trends [2] suggest that with the aggressive scaling of devices the uncertainties due to process variations are expected to worsen in future. The dimensionality of the circuit optimization process will further expand due to an increasing impact of design components affecting the performance and reliability of the circuits. Also, the multi-fold escalation in the design density of the circuits is inevitable. Thus, the circuit optimization methodologies capable of addressing only the problems occurring in current technology generation may not scale well with the next generation issues. The single metric optimization methods that result in a performance shift from one objective to another and are not generalized to incorporate additional metrics are no longer acceptable. Hence, an important challenge in the VLSI circuit optimization is to identify vertically as well as horizontally integrated solution methodologies [14].

Likewise, the existing methods in data clustering are incapable of addressing the clustering requirements for various multi-disciplinary engineering applications. Specifically, these applications require methods capable of simultaneously examining multiple metrics during clustering. Also, a clustering method must satisfy the social fairness [13] from the perspective of each clustering criterion. This would ensure that each clustering metric is satisfied with respect to every other metric in the system.

The motivation for this dissertation is to explore the core issues in these problem domains, and develop new multi-metric optimization approaches that exhibit the following features.

- A framework that is generalized in its ability to incorporate any number of optimization metrics that may be necessary to be optimized for feasible solutions to the problems. Also, the frame-

work should be reconfigurable to enable relative prioritization of the metrics to be optimized as per the requirements.

- A domain independent approach that is easily portable to solve the optimization problems in several knowledge disciplines.
- An approach that is fast, scalable to larger problem sizes, accurate in terms of optimizations, and feasible to solve real problems.
- A method that is capable of addressing the impact of randomness at several avenues. In the context of VLSI circuit optimization this property is extremely important for addressing the impact of process variations in multiple design components.
- An approach that is capable of inherently modeling the multi-objective optimization problems where the objectives are competing or conflicting in nature.
- A methodology that can guarantee the optimization is performed from the perspective of each metric, and hence satisfies the social fairness property.

Several flavors of utilitarian optimization methods have been widely applied to solve the problems in the field of economics and finance [15]. In recent years, computer scientists have explored the realm of utilitarian methods to solve various computer science [16–23] and computer engineering [24–26] problems. The successful implementation of utilitarian approaches in these application areas has benefited the engineering research community. This encouraged us to explore these methods to solve the problems in VLSI-CAD and spatial pattern clustering.

1.2 Why Utilitarian Approaches?

The utility theoretic approaches are attractive as optimization methodologies due to some of unique features and properties that they possess. The two variants of the utilitarian methods are game theoretic optimization techniques and expected utility theory based techniques. Game theory [27, 28] is a microeconomic approach for visualizing a problem as a situation that consists of several players, each player competing with all other players in the system and trying to maximize its own utility or gains from the system. In this competitive setting, an equilibrium point is identified that maximizes the

utility of each player with respect to every other player in the game. Thus, the performance criteria of the systems as a whole are determined by a combination of the performance criteria of the individual agents. The salient features of game theory that serve as reasons for application to the optimization problems are:

- The situations of conflict and cooperation are most effectively modeled as games [29–31].
- Game theoretic models have simple and well defined environments for a variety of problems.
- A methodology like *Nash equilibrium* that identifies a socially fair solution, perfectly complements the problems modeled as a game. The social fairness of the solution is a particularly attractive feature from the perspective of multi-metric spatial clustering.

The expected utility theory [32] was proposed by Von Neumann and Morgenstern in 1944, as a sound prescription for rational decision-making. This theory has been widely studied and applied in the various fields of science and engineering like political sciences, finance, economics, computer networks, and distributed computing. The success of utility theory is attributed to the fact that it enables the designers (or decision makes) to visualize the optimization problems from a different perspective. As a simple example, let us consider a stochastic optimization problem in the mathematical programming setting, where the objectives are to be minimized while satisfying the constraints that are randomized in nature. In the expected utility framework, this optimization problem can be conceived as an optimization problem in which the risk of failure of constraints is minimized by maximizing the expected utility of the constraints. In large scale stochastic optimization problems, like those in VLSI-CAD, this may help in substantially reducing the size of the problem, as well as translating a stochastic optimization problem to the deterministic equivalent under certain situations.

1.3 Scope and Contributions

This dissertation explores the various optimization issues currently existing in the VLSI-CAD field, specifically at the circuit level. It also identifies the concerns for spatial data clustering from the viewpoint of its applications in several multi-disciplinary areas. We identify the different metrics that are required to be examined for pragmatic solutions to these problems. The state of the art research is studied to evaluate the feasibility, portability and scalability of the existing solution methodologies

for next generation technologies and emerging research disciplines. The generalized multi-metric optimization frameworks based upon the utilitarian methods are developed to solve these problems.

The theme of this dissertation and the major contributions are summarized in Figure 1.1. A short description of the research works that contributed to the dissertation is as follows.

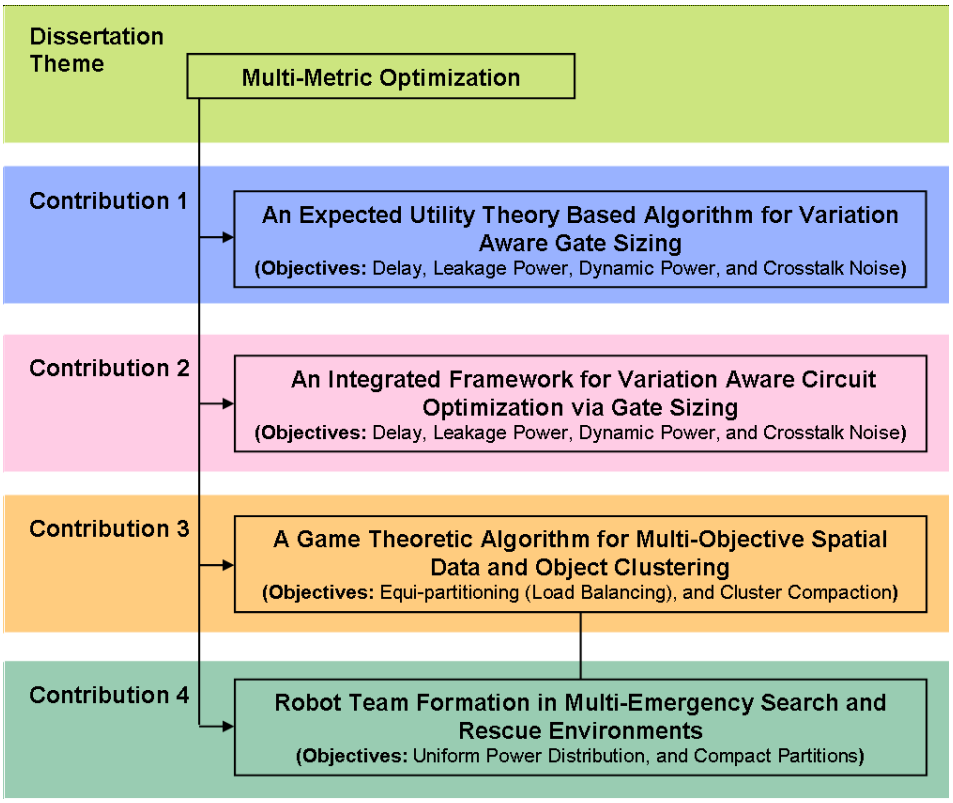


Figure 1.1 Scope and contributions of the dissertation. The theme of the dissertation is to identify and develop new multi-metric optimization methods for VLSI circuit optimization and spatial data and pattern clustering.

- Expected Utility Based Optimization: Multi-objective optimization of delay, leakage power, dynamic power and crosstalk noise in VLSI circuits is performed via gate sizing using a methodology that is based on the concepts of expected utility theory and constraint risk minimization. It identifies a deterministic equivalent model of the stochastic optimization problem using the concepts of bounded rationality. The methodology is variation distribution independent, and identifies solutions with high levels of utility, in the presence of scarce information about the distribution of the process variations. The method is capable of addressing the impact of process variations and randomness at several levels, both in the objective function as well as in the

constraints. This approach effectively tries to minimize the risk of violation or failure of the constraints in the model, evaluated and controlled by an expected utility measure that is maximized to ensure that a constraint is satisfied. The deterministic model identified using this approach is especially attractive for optimization in large scale VLSI-CAD problems.

- Integrated Framework for Circuit Optimization: In this work, a new variation aware multi-metric gate sizing framework has been developed, which can be used to perform optimization of several metrics like delay, leakage power, dynamic power, and crosstalk noise etc. The proposed framework is completely reconfigurable and generalized in terms of its capability to incorporate new metrics and selectively prioritize the metrics depending upon the design requirements, with minimal changes in the model. More importantly, any mathematical programming approach can be utilized within this framework, to solve the optimization problem. The process variation effects are incorporated as stochastic components in the delay model. An important aspect of the proposed framework is the identification of the inter-relationships between dynamic power, leakage power, and crosstalk noise in terms of gate sizes, and modeling them in a unified manner.

- A Microeconomic Approach to Spatial Data Clustering: A novel multi-objective clustering approach that is based on the concepts of microeconomics, specifically game theory, has been developed in this work. This approach is capable of simultaneously optimizing multiple conflicting objectives. The methodology consists of three components, an iterative hill climbing based partitioning algorithm, a multi-step normal form game theoretic formulation, and a Nash equilibrium based solution methodology. The normal form non-cooperative game consists of randomly initialized clusters as players that compete for the allocation of resources (data objects). The Nash equilibrium based methodology evaluates a solution that is socially fair for all the players, and any mathematical hill climbing algorithm can be used to update the clusters after each iteration of the game.

- Robot Team Formation: The rescue robot teams formation problem in the multi-emergency search and rescue environments is a practical application of the microeconomic spatial clustering algorithm being developed. In these environments, robots performing the search and rescue operations in the field are required to be divided into teams since the power dissipation in inter-robot communication and the robot to base station communication is high, while the robots are

running primarily running on batteries, and each emergency location is required to be attended all the time. Dropping out of all the robots servicing a locality would significantly hamper the rescue process. Thus, in this work, robot teams are created on the basis of cluster compaction and uniform power distribution objectives to identify decentralized robot teams with each robot in a team closest to its communication gateway, as well as each team is equally represented in terms of its strength (battery power).

1.4 Outline of Dissertation

The remainder of this dissertation is organized in six chapters. Chapter 2 describes the background and the state of the art research related to the problems being addressed in this dissertation. Specifically, a short tutorial of the important concepts in expected utility theory, mathematical programming, and game theory is presented. Also, the state of the art research in the field of variation aware circuit optimization, and data and pattern clustering is described in details. In Chapter 3, a risk averse utilitarian approach VLSI circuit optimization under scarce information about the process variations is presented. This is a post layout gate sizing approach for multi-metric optimization. Here, the expected utility theoretic methodology is applied to convert the stochastic optimization problem to a deterministic equivalent model. In Chapter 4, an integrated framework is developed for multi-metric optimization of delay, leakage power, dynamic power, and crosstalk noise considering the effect of process variations in the nanoscale VLSI circuits. This gate sizing framework is completely reconfigurable and generalized to incorporate, remove or prioritize the metrics to be optimized. Chapter 5 defines the problem of multi-objective spatial clustering in the context of novel multi-disciplinary application areas, and develops a novel game theoretic clustering algorithm. The different components of the game theoretic modeling are explained in details and the simulations are performed to evaluate the efficacy of the proposed method. In Chapter 6, the problem of robot teams formation in the multi-emergency search and rescue environments is described. The game theoretic clustering algorithm being developed and discussed in Chapter 5 is adapted to solve this problem by forming teams on the basis two optimization objectives cluster compaction, and uniform power distribution. The concluding remarks and the suggested future work in terms of extensions to the problems addressed in this dissertation, and other ideas for further refinements are given in Chapter 7.

CHAPTER 2

BACKGROUND AND RELATED WORK

In this chapter, we present a brief introduction of the various concepts that form the basis for the research described in this dissertation. Specifically, we discuss the expected utility theoretic approach and the game theory approach. These approaches are used to solve the multi-metric optimization problems in the context of VLSI circuit design and spatial pattern clustering problems. The utility theory is used for solving the VLSI circuit design optimization problem, specifically, gate sizing. Game theory is applied in solving the multi-objective pattern clustering problem. Since, stochastic and mathematical programming methods are used in the proposed solution, some background on these topics is provided. We briefly introduce the various VLSI circuit optimization techniques available in the literature, and present issue of VLSI manufacturing process variations effects in the nanometer regime. A detailed discussion of the various related works for VLSI circuit optimization and spatial pattern clustering is also presented in this chapter.

2.1 Utility Theory

A utilitarian theory forms the ethical framework for effective moral action. In this framework, the measure of satisfaction is quantified in terms of the utility of the satisfaction, and is attempted to be maximized by an individual. The utility is often measured as the happiness, as the satisfaction of preferences, or the preference utilitarianism. The philosophy behind the utility theory is to achieve the greatest good for the greatest number. Utility theory has been used as a framework to argue for the value of different actions. Two primary variants of the utility theory in terms of the expected utility optimization exist in the literature. In the first form, the individuals, also known as the agents, try to formulate and act under guidance of rules that maximize the utility if they were to be consistently followed. Alternatively, in the second variant, the goal is to minimize negative utility rather than maximizing the positive utility.

The utility of an action or a state of environment maps the state onto a real number to describe the degree of satisfaction from the state [33]. This notion of the utility has two important implications in terms of the goals. First, the scenarios where the goals are competing or conflicting, the utility function specifies the appropriate trade-off. Second, the situations where several goals are specified, none of which can be achieved with certainty, the utility function maps the likelihood of success of each goal according to the weighted importance of the goals. The overall utility based system can be represented by a simple diagram as shown in Figure 2.1.

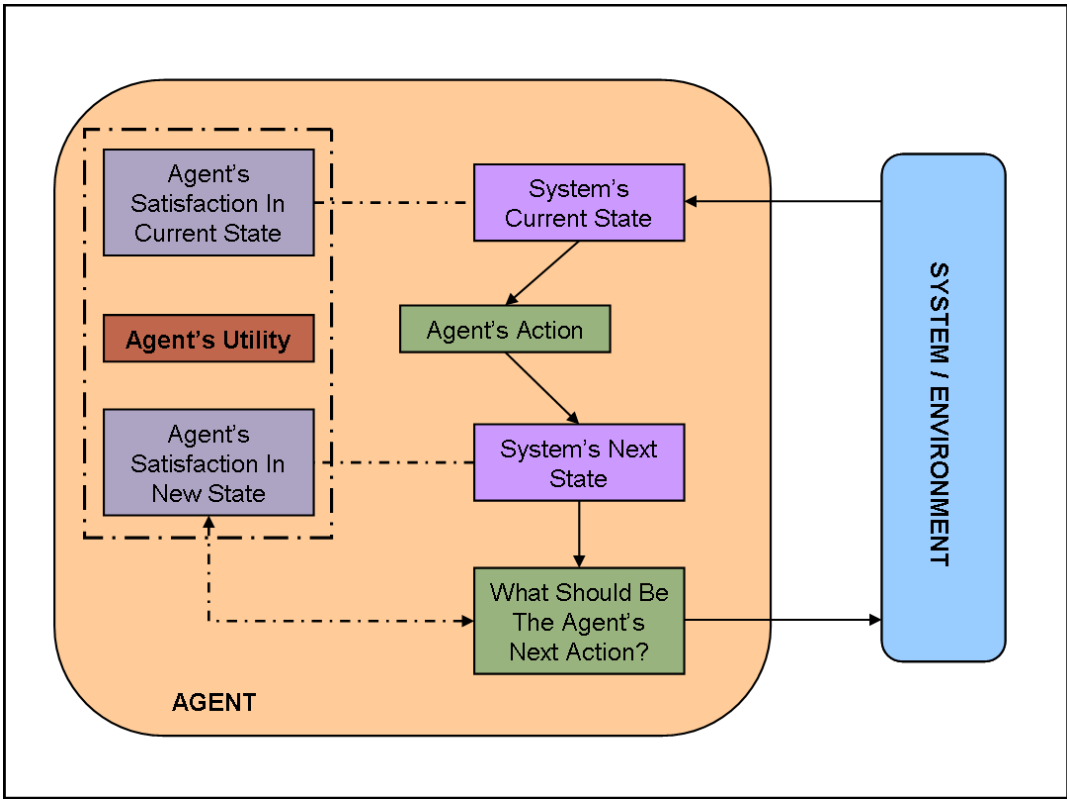


Figure 2.1 A utility based system from the perspective of a rational agent. Depending upon the system's current state, the agent's satisfaction from the current state, and the agent's action, the satisfaction of the agent in the system's next state is identified. The agent chooses its future actions based on the change in its satisfaction value due to its own action in the previous state.

On the basis of the number of rational agents interacting in the system, the utility theory can be categorized as expected utility theory and game theory. In the expected utility theory, the system assumes a single agent playing a game against the nature, whereas in game theory, multiple agents interact with the nature and against each other in an autonomous manner.

2.1.1 Expected Utility Theory and Risk Aversion

In an environment where the agents may not have complete control or access to the environmental variables, a situation of uncertainty would arise. As an example, in the semiconductor devices fabricated with the sub-100nm technology nodes, the environmental factors may affect the manufacturing process significantly, thereby causing inconsistencies in the fabricated devices. The CAD engineers are unaware of the degree of disparity between the specifications and the actual designs. This uncertainty changes the way in which an agent (or designer) makes decisions. In the presence of uncertainty, the actions of the agents shift from deterministic actions to the preferences as a function of the outcome probabilities of the actions. The *expected utility function* maps these preferences to real values.

An action a of an agent A in the expected utility framework would have a set of possible outcomes (also known as states) $O_i(a)$ as a consequence of that action. The index i ranges over the set of outcomes. Also, corresponding to each action a, the agent A assigns a probability $P(O_i(a)|Do(a),K)$ to each outcome. Here, $Do(a)$ is the proposition that the action a results to the associated outcome, given the agent A's information or knowledge K of the environment. The *expected utility* of an action given the knowledge K of the system is given by $EU(a|K)$ as shown in Equation (2.1)

$$EU(a|K) = \sum_i P(O_i(a)|Do(a),K)U(O_i(a)) \tag{2.1}$$

Here, $U(O_i(a))$ corresponds the quantitative measure of the utility of the outcome O_i for the action a. According to the principle of maximum expected utility, the rational agent should choose an action that maximizes its expected utility EU. This notion of utility in terms of probabilities and the outcomes was proposed by John Von Neumann and Oskar Morgenstern in their 1944 book *Theory of Games and Economic Behavior* [32]. According to this theory, if an agent maximizes a utility function that correctly reflects the performance measure by which its behavior is being judged, then it will achieve the highest possible performance for itself.

2.1.1.1 Expected Utility

The utility function maps the states to the real numbers. Hypothetically, the utility of a state could be any real number depending upon the agent's choice, and is an arbitrary function. However, in practice, the preferences of the agents follow a more systematic approach. In a simple economic setup,

the utility can be considered as a monotonic preference function of the monetary values. According to this definition, the utility of the action monotonically increases as the wealth increases in a gambling or a lottery type of situation. However, the utility may not be a linear function of the expected monetary value. This can be explained with a simple example. Suppose, in a game of 'deal or no deal', you have already won $1,000,000. At this stage, the banker asks you if you would like to open one more case that may have $3,000,000. If the case has $3,000,000, you will win the whole amount; otherwise you will go home with no money at all. In such a situation, the expected monetary value of the gamble is $0.5 * \$0 + 0.5 * \$3,000,000 = \$1,500,000$. This value is greater than your current earnings. However, would you be willing to play such a gamble? This is a subjective question, and it depends upon several factors, including your current financial status without the million dollars, the improvement in the life style a million dollars can bring, and how much you value the additional two millions if you already have a million dollars. Thus, utility is not directly proportional to the expected monetary value.

2.1.1.2 Risk Aversion

Risk aversion is intuitively defined as situation where an agent, when faced with the choice of comparable returns, tends to choose the less risky alternative [34]. In an expected utility framework, this concept can be explained through the concave function graph shown in Figure 2.2. Here, X is a random variable which can take on two values, $x1$ and $x2$. Considering p be the probability that $x1$ happens and $(1-p)$ be the probability that $x2$ happens. The expected outcome $E(x) = p * x1 + (1-p) * x2$ is shown on the X axis as a convex combination of $x1$ and $x2$. Considering a $u : \Re \rightarrow \Re$ be an elementary concave utility function, as shown in Figure 2.2, the expected utility is given as $E(u) = p * u(x1) + (1-p) * u(x2)$ denoted by B, between $A = (x1, u(x1))$ and $C = (z2, u(z2))$. Now, by comparing points B and D in Figure 2.2, it is identified that the utility of expected income, $u[E(x)]$ is greater than expected utility $E(u)$, given by,

$$u[p * x1 + (1-p) * x2] > p * u(x1) + (1-p) * u(x2) \tag{2.2}$$

Now, we can consider the scenario shown in Figure 2.2 as two lotteries such that one pays $E(x)$ with certainty and another pays $x1$ or $x2$ with probabilities p and $(1-p)$ respectively. According to the Von Neumann-Morgenstern utility notion, the utility of the first lottery would be $U(E(x)) = u(E(x))$

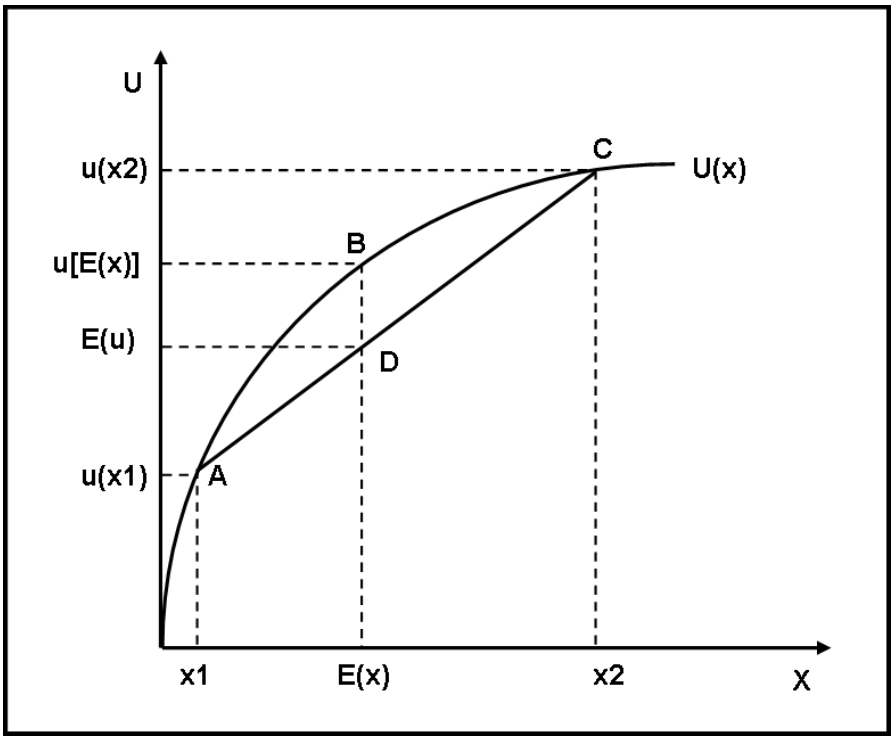


Figure 2.2 A function representing the relationship between expected utility and risk aversion. In a concave utility function, if the average returns for the agent in situations of deterministic decisions and probabilistic decisions are comparable, the expected utility of the risk averse decisions is higher than the risk centric decisions.

received with certainty and the utility of the second lottery would be $U(x1,x2;p,1-p) = p*u(x1) + (1-p)*u(x2)$. In this situation, even when the expected income in both lotteries is same, the obvious decision for a risk averse agent would be $E(x)$ with certainty.

In a VLSI circuit optimization problem under uncertainty, a similar situation arises. The optimization of the performance objectives can be improved by increasing the risk of failure of the timing constraints, thereby resulting in an increase in unreliability of the circuit. Specifically, in this paradigm, the marginal utility declines much more rapidly as compared to the elementary utility function curve as shown in the Figure 2.2. Thus, a stricter notion of *quadratic utility function* can be used in such scenarios. The quadratic utility function [32] is given as:

$$u(x) = \alpha + \beta x - \gamma x^2 \tag{2.3}$$

where α, β and γ are the coefficients of absolute risk aversion, derived to evaluate the utility function.

2.1.2 Game Theory and Nash Equilibrium

Game theory can be defined as a collection of mathematical models formulated to study the situations of conflict and cooperation between intelligent rational decision-makers. Game theory analysis situations in which two or more individuals make decisions that will influence one another's welfare. These decision makers, also known as the *players*, choose from a finite list of alternative courses of actions, leading to well defined outcomes expressed in terms of numerical payoffs associated with the chosen course of action for each decision maker.

Formally, modern game theory began with the publication of the seminal book by Von Neumann and Morgenstern in 1944 [35]. In 1951, John Nash described an equilibrium concept [36] for non-cooperative games as a configuration of strategies that ensures a win-win situation for all decision makers. This concept of cooperation under non-cooperative environments was phenomenal, and as a result game theory has been successfully applied extensively in the field of economics, engineering [25] [24] [16], and several other real life situations of decision making under uncertainty.

The important elements of a game are categorized as players, strategies, strategy sets, strategy combinations, payoffs, information, and equilibrium. The players are a set of rational decision makers, each having a set of strategies $S_i = \{s_i\}$ available with them. A strategy s_i is a rule that a player i uses to choose an action at each instance of the game. Corresponding to each strategy, a utility is associated, which is represented as a payoff denoted by $P_i(s_1, \cdots, s_N)$ that i tries to maximize. A strategy combination is an ordered set $\mathbf{s} = (s_1, \cdots, s_N)$ that consists of one strategy for each of N players, and one such combination that maximizes every player's payoff in the game is identified as an equilibrium point.

The idea behind game theory can be explained with the aid of an interesting and a classical example of *prisoners' dilemma*. Consider a situation where the police has convicted two computer programmers Robin and David in a case of critical data theft from the database of the company that employs them. The police is assured that they are guilty, but they could not prove it since there are no witnesses. So, the police is dependent upon the convicts' testimonies to identify who is guilty. The police decides to keep them in separate rooms for interrogation. The convicts are given only two options, confess or refuse. The police has decided to assign different penalties for the convicts depending upon their

independent responses, as well the combination of the responses of both convicts. The penalties for different scenarios are as follows:

- If both convicts confess to stealing the data, the punishment is 5 years of jail term for each of them.
- If one prisoner confesses and other refuses, then the confessor is given 1 year of jail term for his truthfulness, and the one who has refused is penalized for 10 years of jail term.
- If both convicts refuse to accept their involvement in the theft, then both of them are sentenced for 3 years due to the lack of sufficient evidence.

Now, the situation before the convicts is complex, since they can not communicate and decide what they should be doing. Also, each of them is afraid of the other's position or standpoint. This situation can be modeled as a matrix game as shown in Figure 2.3.

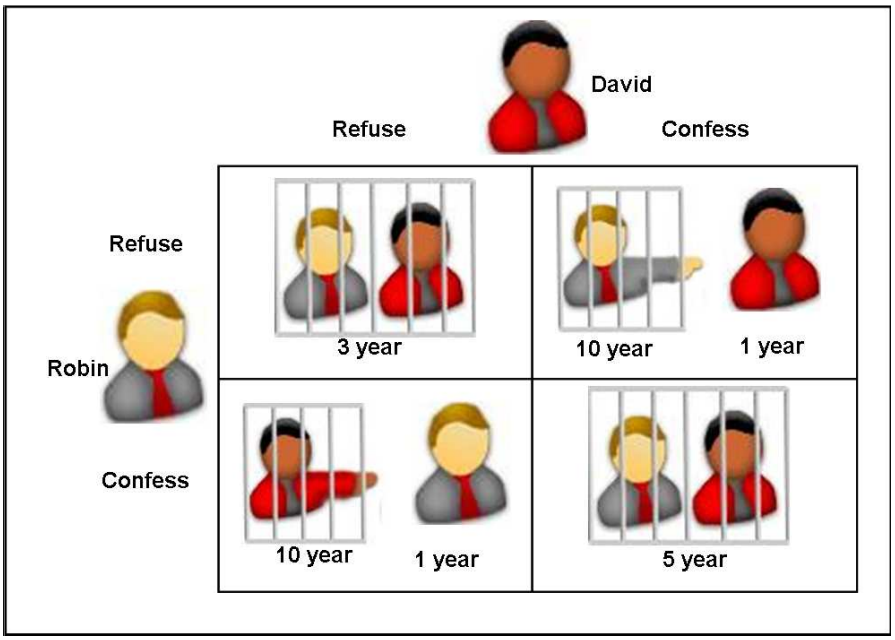


Figure 2.3 A simple example of two player non-cooperative normal form game. The prisoners' dilemma in terms of the strategies (confess, refuse) and the different payoffs (1 year, 3 years, 5 years or 10 years) are shown.

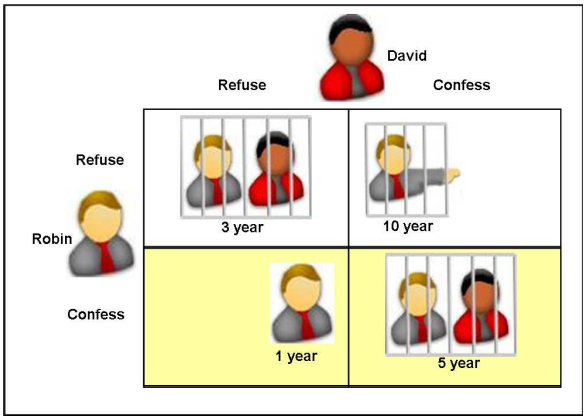
In this example, the two convicts, David and Robin, are the players of the game. Each player has two strategies, confess and refuse. The elements of the matrix game are the payoffs or utilities associated with the strategies chosen by the players. For example, if Robin chooses his strategy of

refusing to be involved in the theft, his punishment will depend upon the strategy chosen by David. If David also refuses, then Robin will get 3 years of jail term, whereas if David accepts their involvement in the theft, then Robin will be sentenced for 10 years in jail. The information available with each player is the strategies available with the other player. A strategy combination is a tuple consisting of one strategy corresponding to each player in the game. One such strategy combination is the set (confess, confess).

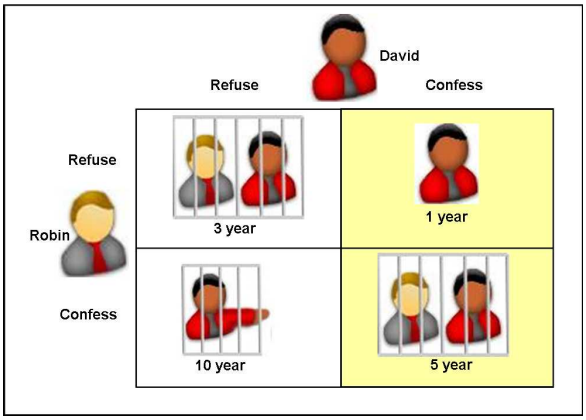
The solution of a game model is identified using an equilibrium technique. Nash equilibrium [36] is one such technique that has been widely used to solve the game theoretic formulations. Nash equilibrium in a non-cooperative game setting is identified as a point (or strategy combination) at which no player can improve its utility by deviating from that point, considering the other players do not deviate from that point. A Nash equilibrium in the prisoners' dilemma game can be explained with the aid of the Figure 2.4(a) – 2.4(c).

As shown in Figure 2.4(a), Robin, if refuses to testify that he was involved in theft, would receive a term of 10 years in the worst case scenario, and 3 years in the best case scenario. However, if he confesses his involvement in the theft, would serve a term of 5 years in the worst case, and 1 year in the best case scenario. Thus, to confess his involvement is the obvious dominant strategy for him. This is shown as the yellow shaded region in the figure. Convict David has the similar situation as shown in the payoff matrix in Figure 2.4(b). With the similar set of arguments, it is logical for David to also confess his involvement in the theft.

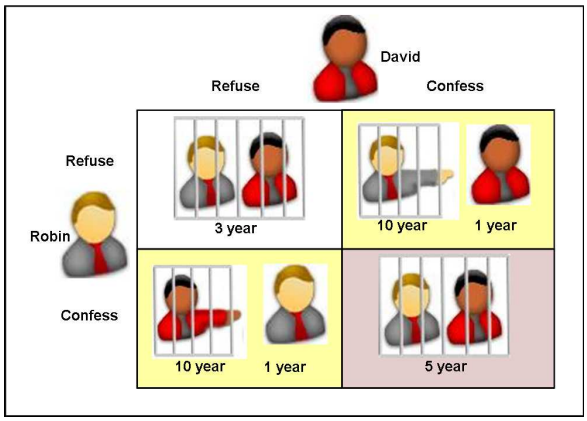
Now, if we take the dominant strategies of both the players, the final equilibrium strategy is identified, as shown in Figure 2.4(c). Here, the purple shaded region denotes the intersection of the dominant strategies of the two players. This point is precisely called the equilibrium point, and the strategy combination (confess, confess) is the Nash equilibrium strategy. At this strategy point, if David tries to change its strategy from confess to refuse, while Robin maintaining his position of confession, David will only loose and will get more years in the jail term. Similar situation occurs when Robin tries to change his strategy unilaterally. Thus, at the Nash equilibrium point each player is satisfied with respect to every other player in the game.



(a) Strategies and Payoffs of Robin



(b) Strategies and Payoffs of David



(c) Nash Equilibrium Strategy Combination

Figure 2.4 Generation of strategy sets, identification of the dominant strategies, and Nash equilibrium in prisoners' dilemma game. In (a) and (b) the respective strategies and the payoffs for Robin and David are shown. The Nash equilibrium strategy on the basis of the dominant strategies for each player is shown in the right bottom box of (c).

2.1.2.1 Classification of Games

Games can be classified on the basis of several different criteria. Some of the important classifications of the games are given as follows.

- Number of players - 2-player (prisoners' dilemma), N-player (finite), and infinite player games
- Number of moves and choices - finite strategy set and infinite strategies
- Degree of opposing interests - zero-sum games and general-sum games
- Degree of cooperation - cooperative games and non-cooperative games
- Number of stages - one-shot games and repeated games

- Time dependence - static games and dynamic games
- Involvement of probability - deterministic games and stochastic games

Here, we will discuss some of these classifications, specifically the ones of our interest in terms of solving the multi-metric optimization problems. To get detailed information on the other classification criteria, the readers may refer to [27, 28]. In a classification based on the degree of cooperation, the non-cooperative games consist of rational players choosing their strategies independently, with nominal information of the strategies available with the other players. Each player plays a strategy that is its best response to the strategy combination of the other players. Unlike cooperative games, the coordination among the players is not forced externally, but is self-enforcing. In multi-player situations, where external communication for cooperation is complex, and hence impractical, the non-cooperative games are pragmatic. Non-cooperative games can be further classified as normal form or strategic games, and extensive form games. In the normal form games, players simultaneously choose their strategies and a strategy combination that gives the best possible payoffs to every player is considered as an equilibrium point. Whereas, in the extensive form games, the players move in a sequential order, and the order of play affects the final outcome of the game. Since all the players make their moves simultaneously in a normal form game, they do not get to learn each other's private information.

2.1.2.2 Mathematical Representation

A non-cooperative normal form game is a finite game if the strategy sets $S_1, \cdots, S_N$ are finite. Here, N is the set of all players in the game, and S_i is the set of all the strategies of player i. The game is being represented as:

$$G = (S_i, p_i); \forall i \in N \tag{2.4}$$

Here, p_i represents the payoff function for player i, and is given as:

$$p_i = \prod_{i \leq N} S_i \rightarrow \Re \tag{2.5}$$

For the game G represented by Equations (2.4) and (2.5), the N-tuple of strategies $s_1^*, \cdots, s_N^*$ where $s_1^* \in S_1, \cdots, s_N^* \in S_N$, is defined as the Nash equilibrium point of G if Equation (2.6) satisfies $\forall s_i \in S_i$

and $i = 1, \cdots, N$.

$$p_i(s_1^*, \cdots, s_i^*, \cdots, s_N^*) \geq p_i(s_1^*, \cdots, s_{i-1}^*, s_i, s_{i+1}^* \cdots, s_N^*) \tag{2.6}$$

Qualitatively, Nash equilibrium is a socially fair, good quality solution point at which every player is satisfied with respect to every other player.

2.1.2.3 Critique of Game Theory

Although game theory has been widely studied and applied in several important application areas, it is often criticized for some of its properties. The primary critiques of game theory are:

- Why non-cooperative games?: If the prisoners' dilemma game being presented above is revisited, a natural question that arises is to why not play a cooperative game? This is intuitive, since in such a scenario, players may come out with a more advantageous strategy combination of (refuse, refuse). The issue with cooperative games is that in such games the players need to make prior commitments for cooperation. The prisoners' dilemma game (represented by χ per say) can be transformed into a cooperative game by a mapping ξ, such that $\xi(\chi)$ is another game that represents the situation existing where, in addition to the strategy sets specified in χ, each player would have some wide range of options for bargaining with the other players to jointly plan cooperative strategies. In such situations the strategy set of each player would explode and the game would potentially become inconceivable. Another reason for not considering a cooperative game solution is the requirement of impartial arbitrator in cooperative games, who could perform pre-play communication with all the players beforehand. In such situations, a considerable amount of time is required for such arbitrations, which is not pragmatic in solving real engineering problems.

- Multiple Nash equilibriums and pareto optimality: The Nash equilibrium for a game theoretic model consists of all the dominant strategies. However, there may be multiple Nash equilibriums in a game, and it is possible that several Nash equilibriums may not be *pareto optimal* [28]. A solution is pareto optimal, if there exist no other solution that can make at-least one individual better off without making any other individual in the system worse off. A good example for such a situation is the Prisoners' dilemma situation. Here, the dominant strategy and the Nash

equilibrium point is the combination where both the prisoners confess their crimes, which is reasonable from the players' as well as the system's perspective, considering that the players are rational and non-cooperative. As evident, the solution is not pareto optimal. The pareto optimal solution point is (refuse, refuse). However, the pareto optimality would require cooperation among the players, existence of focal arbitrator, and a coalition formation, which is infeasible. It is important to note that the criterion of pareto optimality does not ensure that a solution is by any sense equitable and socially fair, which is an important criterion in multi-objective optimization.

2.2 Mathematical Programming

A mathematical programming problem is an optimization problem, wherein one seeks to minimize or maximize a real valued function of real or integer variables, subject to constraints on the variables. Mathematical programming studies the following properties of an optimization problem:

- The mathematical properties of the optimization problem.
- The development and implementation of the algorithms to solve the optimization problems.
- The application of these algorithms to real world problems.

The mathematical programming is primarily performed to solve two types of problems, continuous and discrete. The continuous optimization problems could be constrained or unconstrained. To solve the unconstrained optimization problems, several methods like non-linear programming, non-linear least square optimization methods, non-differentiable optimization methods and other global optimization methods are applied. The constrained optimization problems could be linear, stochastic, non-linearly constrained or bound constrained. Several algorithms have been developed to solve such problems [37]. The deterministic discrete optimization problems are solved using integer programming methods. The stochastic optimization problems, which could be discrete or continuous problems, are harder to solve, since they involve uncertainty.

Stochastic programming is a framework for modeling optimization problems that involve uncertainty. Stochastic programming methods take advantage of the fact that probability distributions governing the data are known or can be estimated. The goal here is to find some policy that is feasible for

all (or almost all) the possible data instances and maximizes the expectation of some function of the decisions and the random variables. More generally, such models are formulated, solved analytically or numerically, and analyzed in order to provide useful information to a decision-maker.

Stochastic programming is a widely studied and applied optimization problem to the real world problems since any real world problem almost invariably includes some unknown parameters. Several algorithms and solution methodologies have been developed to solve the stochastic optimization problems. Chance constrained programming, two stage linear programming, multi-stage linear programming, fuzzy mathematical programming and geometric programming are a few state of the art methods to solve stochastic optimization problems. In general terms the optimization methods in this discipline combine the power of mathematical programming with advanced probability techniques, to attack optimization problems that involve uncertainty. A constraint or presumption in these methods is that the probability distributions of the random parameters are known, and cannot depend on the decisions taken.

2.3 VLSI Circuit Optimization

In the nanometer era, the performance of a VLSI circuit is not only determined by the the delay or the frequency of the circuit alone. The reliability, scalability, power dissipation, energy to perform a function, cost, yield and the time-to-market the chips are also important performance metrics. The optimization of these metrics is thus an essential part of designing robust, reliable and high performance circuits. The persistent push for higher performance and reliability in much more complex designs has led to an increasing interest in the optimization techniques. Circuit optimization primarily involves tuning of various components of a circuit to achieve desired changes in the performance metrics. The components that can be tuned, include transistors, wires, buffers, power supply voltage, and threshold voltage etc. [38]. In addition to these continuous tuning techniques, various discrete optimization methods like buffer insertion, reordering of input pins, and choice of gates from discrete libraries etc. are also widely studied in literature. Since, in this dissertation multi-metric optimization of delay, power and crosstalk noise is being performed, the methods that are effective for the optimization of these metrics are reviewed. It is important to note that the framework for VLSI optimization being de-

veloped in this research is independent of the metrics that can be incorporated for optimization. Other performance metrics can be added in the model with minimal effort.

In a circuit, the maximum delay is defined as the total delay of the longest path (*critical path*) in the design. Some of the prominent techniques for delay minimization include gate sizing, transistor ordering, defining alternative logic structures, buffer insertion, reducing the voltage swing of the gates, and interconnect wire sizing [39, 40]. In a gate sizing technique, the sizes of the gates in the path are adjusted to minimize the delay of the path. The sizes of the gates in the entire circuit or a sub-circuit are adjusted properly according to their capacitive loads for performance improvement. In the transistor ordering technique, the transistors are ordered in a row and oriented in such a way that the sharing of source and drain regions is maximized. This aids in reducing the total diffusion area and the cell widths. Delay of a circuit can also be reduced by carefully replacing logic structures in a circuit. For example, a function like $F = ABCDEFGH$ being implemented using 5 two input NAND gates, 2 two input NOR gates and a NOT gate can be replaced by a eight input NAND gate and a NOT gate. Another effective technique for delay optimization is to insert buffers in order to isolate the fan-in from the fan-out, thereby reducing the load on the critical path of the circuit. In the buffer insertion technique, a series of cascaded inverters are inserted on interconnects between the gates. In a wire sizing technique, the widths of the interconnect wires are sized to reduce the interconnect delays. The techniques like multi-VDD assignment and threshold voltage scaling have also been applied for delay minimization.

Power dissipation in VLSI circuits is primarily due to two components; static power, and dynamic power [41]. The dynamic power dissipation is due to two sources, switching power due to charging and discharging of load capacitance, and short circuit power due to non-zero rise and fall times of input waveforms. The static power or leakage power dissipation occurs when the device is not active. The three components of leakage power are sub-threshold leakage due to current from drain to source, direct tunneling gate leakage due to tunneling of electrons or holes from the bulk silicon through the gate oxide potential barrier into the gate, source and drain or substrate and substrate reverse biased p-n junction leakage. Several techniques have been proposed to reduce these components of power dissipation. For dynamic power reduction, gate sizing, interconnect sizing, clock gating, supply voltage scaling and buffer insertion are primary techniques.

Dynamic power of the circuit is minimized by sizing down the gates in the circuit. However, such sizing technique increases the delay of the circuit. In order to optimize both delay and dynamic power, a path based technique can be applied, where gates in the critical paths are sized-up and the gates in the non-critical paths are sized down. Alternatively, a global optimization can be performed with delay-power trade-off. Wire sizing technique follows a similar relationship. If the width of the wire is increased, the resistance per unit length of the wire decreases. However, the line capacitance increases, consequently increasing the interconnect power. In a clock gating scheme, the clock is masked such that the switching activity of the idle blocks of the circuit is minimized, thereby reducing dynamic power dissipation. This technique also reduces the clock power dissipation. Supply voltage scaling minimizes the switching power dissipation. Since supply voltage has a quadratic dependency on the switching power, the technique is effective. In this method, either the supply voltage of the non-critical part of the circuit can be lowered in a static manner, or the supply voltage can be dynamically lowered depending upon the performance demand of the circuit.

Leakage power minimization at the circuit level can be performed by applying techniques like gate sizing, threshold voltage scaling, transistor stacking and adaptive body biasing. Since the gate size is directly proportional to the average leakage power of the gate, sizing the gate reduces the leakage power of the circuit. Assignment of high threshold voltage to some transistors in the non-critical paths can reduce the sub-threshold leakage. The transistor stacking method inserts extra transistors (sleep transistors) connected in the series with the pull-up/pull-down path of the gates and turns them 'off' during the standby mode. In adaptive body biasing, the forward body bias (FBB) and the reverse body bias (RBB) is applied to vary the threshold voltage of the transistors, thereby turning them off during the passive mode.

The coupling of a quiet line with one or more switching lines induces noise on the quiet line. If the noise is high, the logic of the quiet line may switch causing logic failures. This crosstalk noise can be reduced by applying methods like wire sizing, wire spacing, wire shielding, sizing of the driver gates of the victim and aggressor interconnects, and sizing of the receiver gates of the victim and aggressor nets [42, 43]. If a wire is sized up, the resistance of the wire increases, thereby reducing the coupling effect on it. Alternatively, if the coupled wires are spaced farther, the coupling capacitance between them reduces, consequently reducing the noise on each of them. In the driver gate sizing, if the victim net's driver gate is sized up, the signal strength on the victim net increases, resulting in a decrease in

the coupling noise on itself. The impact is complementary, since an increase in signal strength on the net induces higher coupling noise on the neighboring nets. Similarly, up-sizing the receiver gate of a victim net reduces the noise on the net. However, the effect of receiver sizing is significantly smaller as compared to the driver sizing.

Several state of the art techniques for VLSI optimization being discussed here are effective, and have been successfully applied for optimization of either delay, power or crosstalk noise. However, among these techniques, *gate sizing* is particularly interesting due to several reasons. Gate sizing is a simple, general purpose post-layout optimization approach that can be utilized to optimize all the important metrics like delay, power, and crosstalk noise. It does not require the incorporation of any additional circuitry in the design, and hence incurs minimum overhead. Gate sizing at the post-layout level does not require any circuit re-routing to be performed. Also, driver gate sizing is the most effective technique for crosstalk noise optimization [43]. Thus, we utilize gate sizing as the optimization methodology for multi-metric VLSI circuit optimization, considering process variations.

2.3.1 Process Variations

The aggressive scaling of devices and interconnects, the limitations of the manufacturing processes, and the environmental noise affecting the manufacturing processes, have significantly affected the VLSI design paradigm, resulting in a transition of the design and optimization process from the deterministic to the probabilistic domain [5]. Such effects degrade the quality of the signals and affect the reliability of the manufactured circuits. These process variations occur primarily due to two factors.

- Environmental Factors: This includes the variations in the processing due to the variations in environmental factors like temperature, power supply voltage, humidity, pressure, electromagnetic interference, cosmic rays etc.
- Physical Factors: These include the variations in the electrical and the geometrical parameters caused due to imperfections in processing technologies like photolithography, planarization, metal etching, polysilicon etching etc.

The physical factors can be further classified as die-to-die physical variations and within-die physical variations. The die-to-die physical variations cause the inconsistencies between the different dies, but

are largely uniform within each die. Due to which, these variations are largely independent of the design implementation and are usually modeled using worst-case design corners. The within-die variations are the variations in the device parameters within a single chip. Due to these variations, different devices at different locations on a single die may have different device features. The variations in gate dimensions within a die are an example of within-die variations.

The within-die variations are caused due to three types of defects.

- Random defects: The defects that are caused due to introduction of foreign particles in the wafer during the processing. These defects can be introduced during any step in the manufacturing process, and can result in creation of opens or shorts in the manufactured circuits.

- Systematic defects: These defects occur due to sub-wavelength lithography process, and can be controlled by incorporating tighter control during the processing, and by applying techniques like optical pattern correction.

- Parametric defects: Such defects occur due to variations in the manufacturing process. As the process technology scales down, with the scaling of the device parameters like gate oxide thickness, gate length, interconnect spacing etc., the impact of parametric variations increases rapidly. The relative impact of these defects for different technology nodes is shown in Figure 2.5.

Another aspect of intra-die variations is that these variations exhibit *spatial correlations*, where the devices that are close to each other have a higher probability of having similar device properties than those which are placed far apart. When coupled with the process variations, these correlations can cause prime reliability concerns. Hence, it is essential for the design tools to account for the uncertainties, and design robust circuits that are insensitive to the process variations.

2.4 Variation Aware Gate Sizing

Several approaches for the optimization of delay, leakage power, dynamic power, and crosstalk noise in the presence of device process variations have been proposed in recent years. In this section, we discuss the state of the art statistical static timing analysis (SSTA) based and mathematical programming based approaches for variation aware gate sizing, on the basis of their strengths, as well as

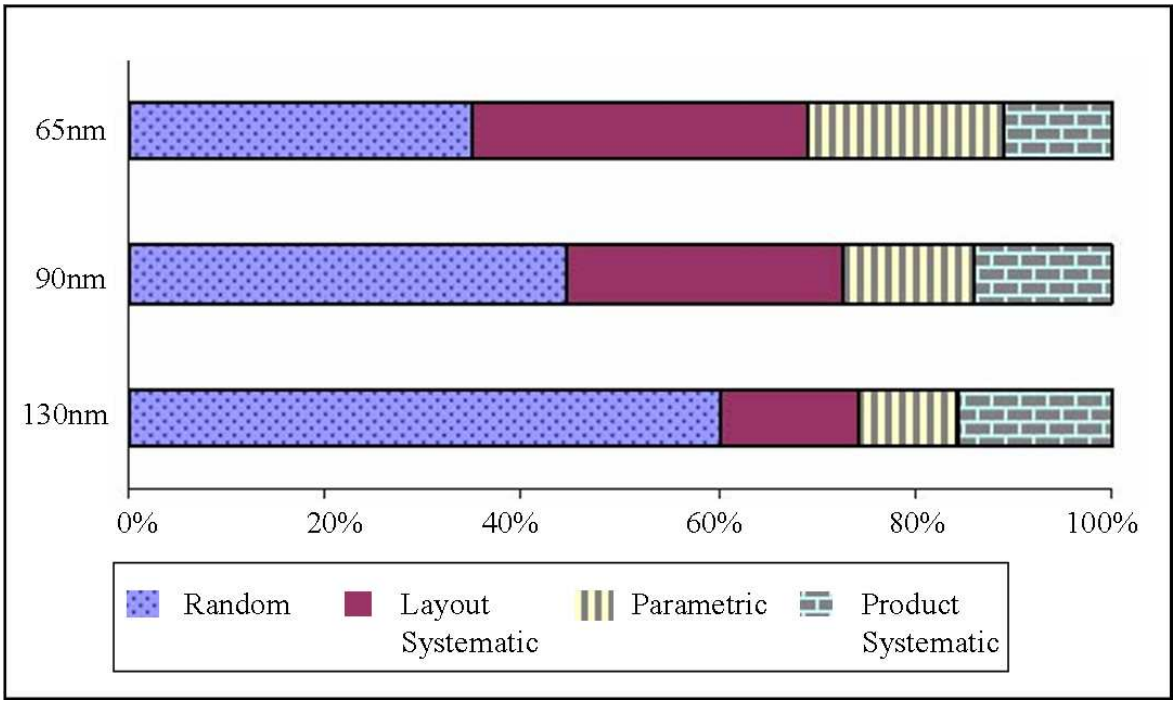


Figure 2.5 Variation impact at different technology nodes [1]. As the process technology is moving toward lower technology nodes, the parametric variations are becoming a dominant factor in determining the total impact of process variations.

limitations at the current technology nodes. The analysis and the next generation VLSI design challenges make a strong case, for identifying new methods for multi-metric circuit optimization of the VLSI design problems.

2.4.1 Optimization Metrics

To analyze and optimize metrics like delay, power, yield, crosstalk noise etc. in the presence of process variations, several methods have been proposed in the literature. Since, this problem is addressed from a gate sizing perspective, the discussion is restricted to review only the variation aware gate sizing methods. Gate sizing is a simple yet effective technique for circuit optimization at the post-layout level, where-in the objective is to identify the optimal drive strength of each gate in the design. In Figure 2.6, a taxonomy of the recent works in gate sizing, classified according to the optimization metrics and the methodologies is presented.

- Power Optimization: Several works can be found in the literature on power optimization with gate sizing, such as minimizing leakage power [44–46], dynamic power [47,48], and total power

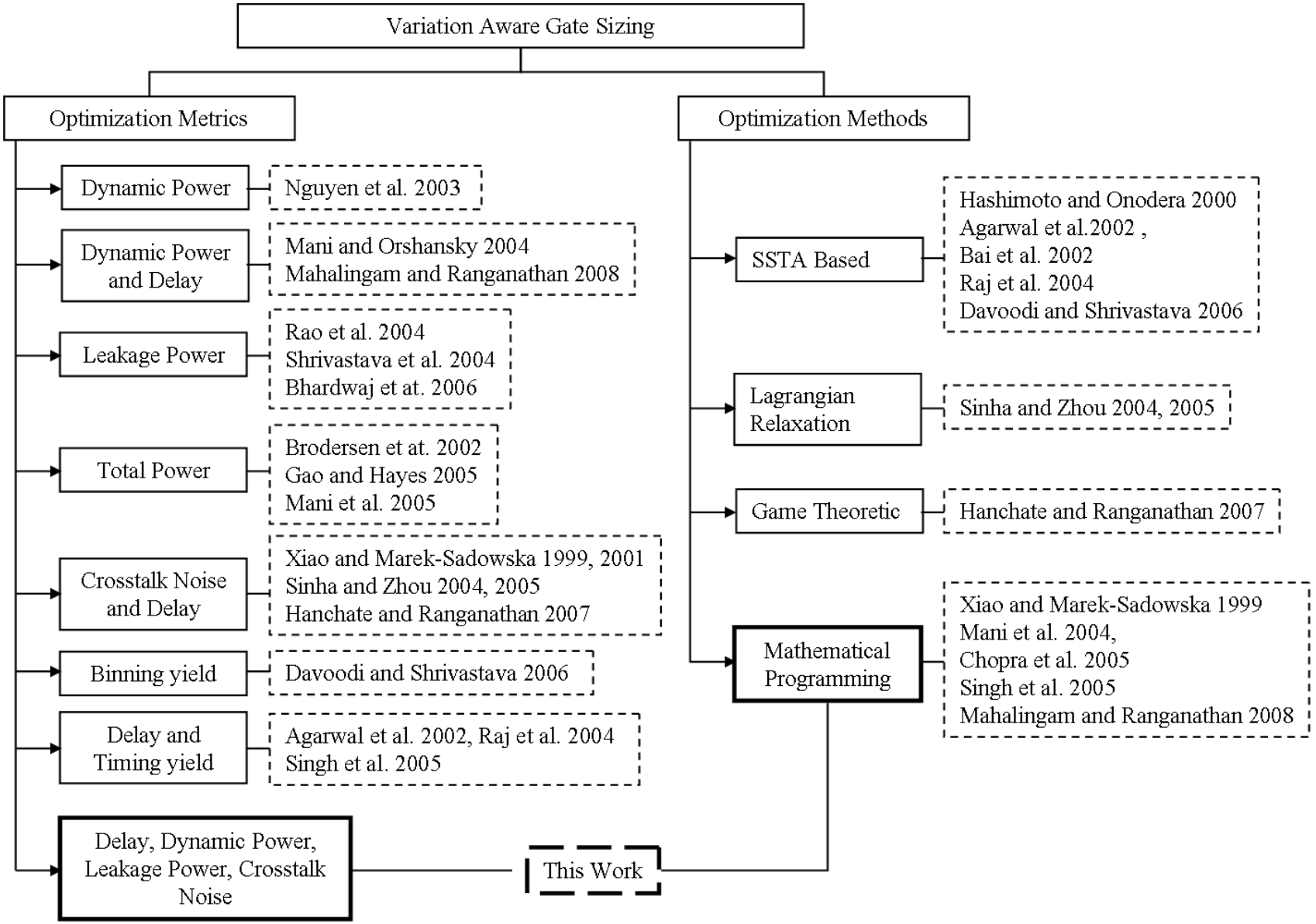


Figure 2.6 Taxonomy of the variation aware gate sizing works. The various works on the variation aware gate sizing are classified on the basis of optimization metrics and optimization methods.

[49–52]. In [47], a dynamic power minimization method is proposed with dynamic power identified as a function of the gate sizes in a stochastic programming model. Similarly, in [48], the authors have proposed a fuzzy mathematical programming based solution for dynamic power optimization. Leakage power minimization under process variations is performed using SSTA based methods [44, 45], in which continuous distributions are propagated through the paths instead of the deterministic values to find the closed form expressions for performance. In [46], a method to estimate the leakage current variation due to inter-die and intra-die gate length variations is presented.

- Crosstalk Noise Optimization: The power optimization methods are primarily single metric models that do not consider the effect of gate sizing on other metrics such as crosstalk noise of the circuits. At the post-layout level, interconnect coupling effects can worsen the signal strength, leading to logic failures. Several techniques to reduce crosstalk noise have been presented in the recent years. In [42], the authors propose a linear programming based formulation for transistor sizing to minimize crosstalk noise in circuits. In another approach [53,54], an yield driven Lagrangian Relaxation based method identifies the upper-bound on noise for each net as a noise constraint. The gates are iteratively sized-up to satisfy the timing and noise constraints, and a simple linear model is evaluated for crosstalk noise minimization. In a recent work [55], a stochastic game theoretic algorithm for post layout delay uncertainty and crosstalk noise optimization considering spatial correlations [56, 57] is proposed. The non-linear crosstalk noise model used in this method is derived from [58], which accurately identifies a closer approximation of the crosstalk noise.

- Delay Optimization: Additionally, the optimization of other important metrics like delay, timing yield and binning yield have also been discussed widely [59–61]. However, this research is largely one-dimensional in the sense that these methods typically aim at optimizing specific metrics and often do not consider the fact that optimizing one metric may negatively impact the optimization of other metrics, leading to an inaccurate analysis of the complete design.

2.4.2 Optimization Methods

Several *SSTA based approaches* have successfully been applied for delay minimization or yield improvement problems [57, 62–64]. These approach intuitively model delay and yield optimization problems in a simple model. The SSTA based approaches, improve over the pessimistic worst-case corner based modeling [61] by performing a mean-variance analysis for the total circuit delay. However, such approaches are essentially path based [65], and traditionally applied to optimize a single parameter. An assumption in a SSTA based technique is that the complete information about the variation distribution of the design parameters is known, and the methodology is based on such assumptions. Several works [6, 7] have assumed a Gaussian distribution. However, global sources of variation follow a log-normal distribution more closely [8, 9] as compared to the Gaussian distribution.

Mathematical programming based approaches have been widely investigated in the literature for optimizing several metrics. An important aspect of mathematical programming approaches for circuit optimization is that any path based problem can be easily converted to the node based equivalent with some sub-optimality being introduced. A geometric programming (GP) approach has been proposed in [66] for delay optimization in the presence of process variations. Although, the approach is robust, the objective function and the constraints are required to be posynomial functions. Thus, modeling a generalized optimization problem in a GP framework requires converting each optimization function and the constraints in a posynomial form, and the problem can only be modeled for minimization of objectives.

In another approach for dynamic power minimization under delay constraints [47], the problem is modeled as a chance constrained stochastic program (CCP). Although CCP techniques can transform simple problems to their deterministic equivalent models, the transformation is extremely difficult for large scale problems. Also, the method is bounded by continuous distributions, and requires a number of operations to be performed iteratively at each node, thus involving higher run times. However, if the variation distribution information is available, the methodology can be modified to incorporate multiple metrics for optimization. Alternatively, the stochastic programming based statistical optimization techniques are reasonably fast, but more conservative in terms of yield, and hence provide lesser savings in terms of objective function optimizations. In a recent work [48], the dynamic power optimization problem considering process variations has been modeled in a fuzzy optimization frame-

work. Here, the stochastic parameters are modeled as fuzzy numbers, and a crisp non-linear problem is formulated to maximize the variation resistance (tolerance) of the circuit. The problem is then solved using commercially available optimization solvers. These methodologies typically aim at optimizing specific metrics and often do not consider the fact that optimizing one metric can negatively impact the optimization of other metrics, leading to an inaccurate analysis of the complete design. The Lagrangian relaxation based methods [53] are limited to either up-sizing, or down-sizing the gates for the optimization.

A shortcoming in the proposed methods for gate sizing considering process variations arise from the fact that several methods [43, 47, 48] incorporate the effect of *process variations* due to only one design parameter, like gate sizes (due to channel length, and oxide thickness). The impact of interconnect variations, which can cause 12-25% variations in the timing of the circuit, depending upon the design and implementation [67, 68] can not be ignored at the deep sub-nanometer level. The process variations can be modeled more accurately using complex and non-linear models that incorporate more parameters, and have higher accuracy [55]. The disadvantage of such a modeling lies in the implementation complexity.

2.5 Spatial Data Clustering

Spatial data clustering involves the grouping of objects into a set of sub-groups in such a manner that the similarity measure between the data objects within a sub-group is higher than the similarity measure between the data objects from different sub-groups. The object and data clustering techniques find applications in a wide variety of scientific disciplines such as biology, computer vision and pattern recognition, communications and computer networks, and information systems. As a result, cluster analysis has received significant attention, and several customized clustering methodologies have been developed to satisfy specific application requirements [11, 12].

2.5.1 Clustering Techniques

Object clustering is a well researched problem reported extensively in the literature, including several detailed survey papers. Jain *et al.* [11], and Scheunders [69] review clustering methods from pattern recognition and image quantization viewpoint, while Kolatch *et al.* [70], and Berkhin [71]

identify methodologies from the data mining perspective. Similarly, Murtagh [72], and Baraldi [73] surveyed various hierarchical, and fuzzy and neural clustering algorithms respectively. For a detailed discussion and survey of different surveys, one is referred to [12].

Clustering techniques can be classified on the basis of several criteria, such as the principles, type of data, shape of clusters, form of final partitions, distance measure, and the number of objectives. Here, we will limit the discussion to partitioning of data sets on the basis of clustering objectives. The three major groups of clustering objectives are compaction, connectedness, and spatial separation. The compaction objective attempts to identify clusters with minimum intra-cluster variation. The KMeans algorithm [74] is the simplest and the most widely mathematical method used in this category. Other algorithms include average-link agglomerative clustering [75] and model based approaches [76]. Clustering with an objective of maximization of connectedness ensures that neighboring data items share the same cluster. The density-based methods [77], and single-link agglomerative clustering methods [75] implement this principle to identify clusters with arbitrary shapes. In spatial separation based methods, the objective is to maximize the inter-cluster separation. However, it provides little guidance during clustering and may produce trivial results. Additionally, an important criterion that has received significant attention recently in the domain of data clustering is equipartitioning or load-sharing [78]. Load-sharing methodologies have been widely researched in the field of distributed systems [79, 80], but did not receive much attention in clustering domain until recently. The new application domains like ad-hoc networks [81,82] and emergency resource deployment require clusters with almost equal number of data objects per cluster to satisfy the constraints.

From the clustering methodologies perspective, several heuristics based techniques have been developed in addition to the mathematical clustering methodologies. This includes simulated annealing [83], evolutionary algorithms [84–86], tabu search [87], and ant colony optimization [88]. Also, hybrid approaches that combine different algorithms have been proposed in literature [85] [84]. Such techniques are primarily used for feature selection in unsupervised classification, and are largely limited to single objective optimization. The multi-objective clustering problem has been solved using the following principles.

- Ensemble methods: Here, the initial ensembles are created by clustering the data either multiple times using the same algorithm (with different initializations or using bootstrapping) or

using complementary clustering techniques [89]. Then, the solutions are combined to create ensembles using expectation maximization or graph based approaches [89]. However, such *a posteriori* integration of single objective clustering results do not exploit the real strength of simultaneous multi-objective optimization.

- Pareto optimization: A feasible solution is pareto optimal if there is no other feasible solution that is strictly better. Multi-objective pareto optimization [86, 90] performs simultaneous optimization of complementary objectives, and hence, is better than the ensemble based methods.

- Microeconomic methods: The situations of conflicting objectives can be naturally modeled in a game theoretic setting. The problems can be modeled in a framework consisting of players with conflicting objectives competing to optimize their utilities [27,28]. The game is solved using the Nash equilibrium based methodology that identifies a socially fair solution. The social fairness ensures that every player is satisfied with respect to every other player.

Microeconomic approaches have been applied to a wide spectrum of problems in the domain of computer science. Murugavel *et al.* [25] developed auction theoretic algorithms in VLSI design automation for simultaneous gate sizing and buffer insertion problem. Hanchate [24] applied game theoretic concepts for simultaneous optimization of interconnect delay and crosstalk noise through gate sizing, while Gupta and Ranganathan [16] implemented game theory for resource allocation and scheduling in the field of multi-emergency management. In grid computing, negotiating agents have been used for leasing of resources using such models [21, 22]. Similarly, Grosu *et al.* [23, 91] used cooperative games and the Nash bargaining solutions for load balancing in distributed systems, and Lazar [20] implemented auctions for optimal bandwidth allocation in wired and wireless networks.

CHAPTER 3

EXPECTED UTILITY BASED CIRCUIT OPTIMIZATION

Aggressive technology scaling has adversely affected the circuit optimization process in two important ways. The impact of process variations in several components, coupled with multi-fold increase in the design complexity has resulted in a situation that requires the circuit optimization techniques to possess important features like accuracy of optimization, incorporation of process variation effects due to various sources in a single model, and fast execution time. Also, in contrast to the optimization techniques that are based on specific parametric variation distributions (like Gaussian), these circuit optimization techniques should be variation distribution independent. In this chapter, we present a novel approach for circuit optimization in the presence of *scarce information* about the distribution of the process variations. This algorithm relies upon the concepts of utility theory and risk minimization for multi-metric optimization of delay, dynamic power, leakage power, and crosstalk noise, through the gate sizing technique. An important contribution of this work is the identification of a *deterministic linear equivalent* model from a fundamentally stochastic optimization problem, ensuring high levels of expected utility. The algorithm achieves significant speedup in the optimization process for large circuits. This algorithm can address the impact of process variations at several levels including device variations, interconnect variations etc., and is independent of the underlying variation distribution. Using the concepts of bounded rationality, this method minimizes the risk of constraint shortfall in a linear programming setup. The experimental results indicate that the algorithm is efficient, and a comparative study with an existing gate sizing technique shows that our method is multi-fold faster as well as comparable in terms of the optimization results.

3.1 Issues in Circuit Optimization

The scaling of process technology in sub-nanometer regime, and the apposition of Moore's law [92] has affected the realm of CMOS design and optimization process. Due to the aggressive tech-

nology scaling, the impact of device process variations on the design process has aggravated, and consecutively, reliability and performance of the fabricated circuits have degraded. One reason for such an effect is that, at the lower technology nodes, the parametric variations in other design parameters have sizable impact on the circuit performance. For example, in sub-65nm designs, in addition to the gate size variations (oxide thickness, channel length), the variations due to interconnects and vias have sizable impact on the design. Due to these variations, the VLSI design optimization process has switched from the deterministic domain to the stochastic domain in the sense that the sizes of gates, wires etc. are no longer a deterministic quantity, but rather, a distribution. The state of the art research in recent years has addressed the circuit optimization process primarily through the statistical static timing analysis (SSTA) based approaches and mathematical programming approaches.

Various SSTA based approaches are variation distribution dependent, and several works have considered the variation sources of components as Gaussian distribution [6, 7]. However, this assumption has been invalidated by some recent analyses [8], according to which the process variations due to different design parameters follow different distributions. For example, in [93] the authors have identified that the global sources of variation follow a log-normal distribution more closely as compared to the Gaussian distribution. Hence, new methods for circuit optimization that are independent of the underlying variation distributions need to be explored.

Another aspect of the rapid progress in the fabrication technology is the multi-fold increase in the density of the VLSI circuits, resulting in larger and more complex designs. This issue, although independent, has a coupling effect with the process variation impact in the sense that it further worsens the circuit optimization process. The stochastic optimization techniques are inherently slower than their deterministic equivalents for obvious reasons. This is aggravated by the ever-growing size of the designs, and presents the designers with a challenge of identifying optimization methodologies that are faster, can address the effects of process variations, and are yield efficient.

In the circuit optimization domain, the optimization of a single metric may introduce some sub-optimality in the values of other metrics. Although, at higher technology nodes, the impact may be negligible, however, such assumptions are not true for nano-scale designs. Thus, a single metric optimization that results in a performance shift from one metric to another is not practical at this level. As a simple example, if an optimization is performed with an objective of crosstalk noise minimization,

the resulting design may not be low power dissipating. Thus, another challenge in circuit optimization is the quest for methods and solutions that are vertically as well as horizontally integrated [14].

The study of the existing research in circuit optimization at the post layout level, as discussed in Section 2.4, raise similar issues that need to be addressed in developing next generation optimization methods. These can be summarized as follows:

- Most of the works perform a single metric optimization of either delay, power, or crosstalk noise. However, such optimizations are no longer adequate for nanometer designs, and new modeling techniques for multi-metric optimization are required to be developed.

- Several methods assume the process variations to follow certain distributions and are developed to work specifically with those distributions. However, such assumptions are not valid for several variations sources. An important aspect of the next generation VLSI optimization is to identify methods that are variation distribution independent.

- The recent analysis on the variation distribution of the various variation sources identify that the variations do not follow the same distributions as were identified in the preliminary analysis. Thus, the methods that are capable of performing optimization under scarce information about variation distribution are desirable.

- Due to the increasing complexity and size of the VLSI circuits, the circuit optimization process has become slower. Due to the process variations, the optimization process has transitioned from certainty domain to the uncertainty domain, adversely affecting the optimization time. Thus, an accurate and fast stochastic optimization technique that could incorporate the impact of process variations through a simple yet effective modeling is required to be developed.

- With the increasing integration density, the sizes of the circuits are increasing significantly. A fast optimization method is thus required for practical solutions to the large scale VLSI design optimization problems.

- With the scaling of technology, process variations in other components of design are rapidly becoming evident. A modeling technique that can address the impact of process variations at various levels, without complicating the modeling would scale well for the next generation

circuit optimization problems. A generic circuit optimization model capable of incorporating the impact of parametric variations due to several factors is thus desirable.

In this work, we develop a novel expected-utility theory based methodology for optimization of multiple performance metrics through gate sizing technique. This approach effectively tries to minimize the risk of violation or failure of the constraints in the model, evaluated and controlled by an expected utility measure that is maximized to ensure that a constraint is satisfied. The modeling assumes the availability of limited information about the system, i.e. only the mean, and standard deviation of the process variation parameters is available, and not the actual distribution. A linear programming model is identified using these values, and is solved for optimal solution. This methodology is capable of coping with the scant information, evaluates a deterministic equivalent model which is important for large scale problems, and can address the variability in several modeling parameters. The key contributions of the algorithm are:

- Using the concepts of constraint risk aversion and minimization, it yields a deterministic equivalent of the inherently stochastic optimization problem, while ensuring high utility levels.
- Performs optimization in the presence of scarce information about the variation distribution. In terms of scarce information, only the mean and the standard deviation, and not the complete information about the underlying distribution are required.
- Performs simultaneous optimization of multiple metrics. The metrics considered in this work are delay, leakage power, dynamic power and crosstalk noise. The inter-relationship between these metrics in terms of gate sizes is identified and modeled in a mathematical programming model.
- Incorporates the impact of process variations due to gate sizes as well as interconnects.

The resulting deterministic problem is significantly faster than the corresponding stochastic problem, and achieves high timing yields. Also, high level of utility is obtained by controlling the risk from each constraint in the model. The process variation effects and the randomness can be incorporated in the model at various levels including the variations in the gate sizes within the delay models, the interconnect variations, as well as the variations in objective functions. The impact of spatial correlation is also modeled in the optimization methodology using a grid based correlation model [62].

The rest of the chapter is organized as follows. The detailed description of the expected utility based deterministic modeling of a general stochastic optimization problem is presented in section 3.2. In section 3.3, delay, power, and crosstalk noise models used in this work are briefly visited. The models for delay and power have been adapted from literature, while a novel crosstalk noise model has been developed in this research, and is discussed. Also, a relationship between the models in terms of gate sizes is derived in this section. Section 3.4 presents the details of the transformation of stochastic gate sizing problem to the equivalent deterministic model. Experimental results for different scenarios and sensitivity analysis of the algorithm parameters are discussed in section 3.5.

3.2 Expected-Utility Based Modeling

In this section, the methodology to convert a stochastic optimization problem to a linear deterministic equivalent using the concepts of expected utility maximization is presented. In this algorithm, different possible scenarios for a random constraint satisfaction are analyzed in terms of the quadratic utility function. The problem is then converted to a utility maximization constrained deterministic model.

A general stochastic optimization problem is given by (3.1), subject to the random constraint (3.2), along with the set of non-random constraints, and the non-negativity conditions.

$$min\ Z = \sum_{j=1}^{n} z_j s_j; \forall s_j \in S \tag{3.1}$$

$$s.t.\ a_i = \sum_{j=1}^{n} a_{ij} s_j \geq b_i; \forall i \in M \tag{3.2}$$

Here, s_j is the jth design parameter to be optimized, z_j is the weight (unit cost) of s_j, which by itself could be a random value. $a_{ij}s_j$ is a random ith constraint corresponding to the parameter s_j and, b_i is the random constraint satisfaction value. S is the total number of design parameters to be optimized, and M is the total number of random constraints in the problem.

Now, from (3.2), the critical random variable for the random constraint i can be defined as:

$$\eta_i = (a_i - b_i)/\bar{b_i}; \forall i \in M \tag{3.3}$$

where, $\bar{b_i}$ is the mean value of all b_i's. Taking the first and second moments of η_i, we get the mean for the constraint i as:

$$\eta_i = (\sum_{j=1}^{n} \bar{a_{ij}} s_j - \bar{b_i})/\bar{b_i} \tag{3.4}$$

and the variance as:

$$\sigma_i^2 \leq (1/\bar{b_i})^2 (\sum_{j=1}^{n} \sigma_{ij} s_j + \sigma_{b_i})^2 \tag{3.5}$$

where, σ_{ij} and σ_{b_i} are the respective standard deviations of a_{ij} and b_i.

An effective way of controlling the risk of failure of a constraint is by maximizing the expected utility of the constraint. The assumption of scarce information states that only the mean and the standard deviation values, and not the complete information about the distribution for each random variable are available. Also, in the context of scarce information, it is assumed that the *risk of failure significantly exists*, i.e. the negative value of η_i can occur with significant probability, and thus our goal is to minimize that by maximizing the utility value. In the context of gate sizing problem, it corresponds to the situations where the delay constraints are not met due to the variation effects, and consequently affecting the yield. For the justifications and the detailed descriptions of the technical information that follows, please refer to Ballestero's paper [94].

The decision maker's utility can be given by the standard quadratic Von Neumann and Morgenstern utility function [32] for η_i as:

$$U_i(\eta_i) = \alpha + 2\beta\eta_i - \gamma\eta_i^2 \tag{3.6}$$
$$\bar{\eta_i} > 0; \forall i \in M$$

where α, β, and γ are the parameters to be determined. In (3.6), there are three possible conditions:

- *A shortfall*, where $\eta_i < 0$ and the constraint is not satisfied
- *A zero shortfall*, where $\eta_i = 0$, and reflects a critical situation where the constraint may or may not satisfy depending upon the randomness of η_i
- *A surplus*, where $\eta_i > 0$, and the constraint is safely satisfied.

These possibilities and their utilities are pictorially described in Figure 3.1.

Now, for a shortfall, the utility (3.6) decreases rapidly as the η increases, but the value is still positive till $\alpha > 2\beta\eta_i - \gamma\eta_i^2$. This interval is shown as 'bearable shortfall' region in Figure 3.1. So, the *greatest bearable shortfall* η_i^*, which is a very small value, can be expressed as a function of the mean value of η_i ($\bar{\eta_i}$), given by

$$U_i(\eta_i^*) = \alpha + 2\beta\eta_i^* - \gamma\eta_i^{*2} \tag{3.7}$$
$$\alpha + 2\beta\lambda_i\bar{\eta_i} - \gamma\lambda_i^2\bar{\eta_i}^2 = 0$$

where λ_i is a positive parameter close to zero, since the greatest bearable shortfall is a very small value. From the first derivative of utility (3.6) with respect to η_i, we get

$$U_i^{'}(\eta_i) = 2(\beta - \gamma\eta_i) > 0 \tag{3.8}$$

As the utility monotonically increases with η_i (which is negative), less shortfall is preferred to more shortfall.

In case of zero shortfall, the utility is given by α. At this critical point, the randomness of η_i decides if the constraint is met or not. A surplus is thus preferred by the decision makers since the zero shortfall is a random value at the edge. So, a *security margin* in terms of small surplus is preferred. However, a large surplus value is not good since it can adversely affect the achievement of the objective goals. The security margin is shown as the shaded region in the Figure 3.1. A utility maximization function is derived by substituting the values of α, β, and γ in (3.6).

Now, if the first derivative of utility given in (3.8) is equated to zero for maximal value of η_i, we get,

$$\eta_{imax} = \psi_i = \beta/\gamma \tag{3.9}$$

Substituting the values in (3.7), we get

$$\beta/\gamma = \lambda_i^2\bar{\eta_i}^2 + 2\psi_i\lambda_i\bar{\eta_i} \tag{3.10}$$

$$\alpha/\beta = (\lambda_i^2\bar{\eta_i}^2 + 2\psi_i\lambda_i\bar{\eta_i})/\psi_i \tag{3.11}$$

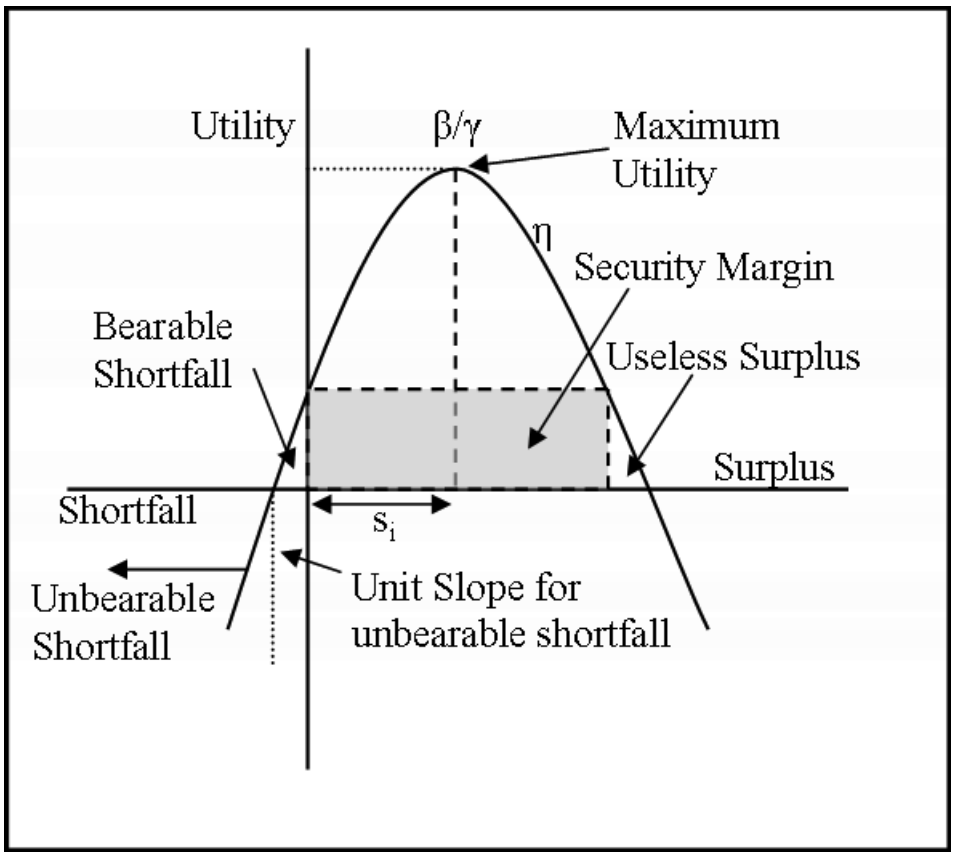


Figure 3.1 Utility curve for a random function. The possible scenarios of shortfall, zero shortfall and the surplus are shown here. The shaded region shows the security margin for random function η, whose value is maximized when the first derivative of the expected utility function is equated to 0.

These values correspond to the greatest bearable shortfall, the mean, and the surplus of minimum utility. Now, the overall maximum utility value can be derived with respect to the mean value as

$$U_{imax} = K_i\bar{\eta_i} \approx \bar{\eta_i} \tag{3.12}$$

The value of K is irrelevant here, since the utility value is just an index. Now, Putting the values of α, β, and γ in (3.6), we get the maximum utility as,

$$\frac{\bar{\eta_i} - \alpha}{\alpha} = \frac{\psi_i^2}{(\lambda_i^2\bar{\eta_i}^2 + 2\psi_i\lambda_i\bar{\eta_i})} \tag{3.13}$$

Now, for a problem in which the shortfall is unacceptable, such as gate sizing, the utility for shortfall is 0. Thus, in Figure 3.1, the bearable shortfall area limits to 0, and hence $\alpha = \lambda_i\bar{\eta_i}$. Now,

substituting this value in (3.13), we get

$$\psi_i \approx 2\bar{\eta_i}; as\, \lambda_i \rightsquigarrow 0. \tag{3.14}$$

In the optimization problem, the expected utility (EU) for each random constraint a_i is to be kept at a high level to assure that the solution points are identified by satisfying the constraints. This is mathematically expressed in terms of first and second derivatives as:

$$\begin{aligned} EU_i(\eta_i) &= U_i(\bar{\eta_i}) + 0.5U_i''(\bar{\eta_i})\sigma_i^2 \\ &= \alpha + (2\psi_i\bar{\eta_i} - \bar{\eta_i}^2)\gamma - \sigma_i^2\gamma \end{aligned} \tag{3.15}$$

A parameter ω, symbolizing the utility value (an index) can be introduced here such that:

$$\alpha + (2\psi_i\bar{\eta_i} - \bar{\eta_i}^2)\gamma - \sigma_i^2\gamma > \omega\lfloor\alpha + (2\psi_i\bar{\eta_i} - \bar{\eta_i}^2)\gamma\rfloor \tag{3.16}$$

where ω is close to unity. The constraint (3.16) ensures that the expected utility of the constraint is close to unity, and is satisfied.

Now, solving (3.16) for σ_i^2, we get,

$$\begin{aligned} \sigma_i^2 &< (1-\omega)(3 + \lambda_i^2 + 4\lambda_i)\bar{\eta_i}^2 \\ &\leq 3(1-\omega)\bar{\eta_i}^2; \bar{\eta_i} > 0 \end{aligned} \tag{3.17}$$

This equation gives a clear relationship between the variance and the mean in terms of expected utility. These relationships are then utilized in identifying a deterministic model, as discussed next.

3.2.1 Deterministic Modeling

In a mean-variance approach for the expected utility maximization problem, the general minimization problem described in (3.1) is converted into an equivalent maximization function of the expected utility, subject to the parametric variance constraints. The expected utility maximization function can

be given as:

$$max\ \ \Lambda = C - Z = C - \sum_{j=1}^{n} z_j s_j \tag{3.18}$$

Here, C is a large positive constant. Also, $\bar{Z} = \sum_{j=1}^{n} \bar{z_j} s_j > 0$, since no resource is free. Thus, the equivalent minimization problem becomes,

$$min\ \ \sum_{j=1}^{n} \bar{z_j} s_j \tag{3.19}$$

subject to the following constraints,

- Constraints (3.17), as developed earlier,

$$\sum_{j=1}^{n} \sigma_{ij} s_j + \sigma_{b_i} < \sqrt{(1-\omega)(3+\lambda_i^2+4\lambda_i)}\,(\sum_{j=1}^{n} \bar{a_{ij}} s_j - \bar{b_i}) \tag{3.20}$$
$$< \sqrt{3(1-\omega)}\,(\sum_{j=1}^{n} \bar{a_{ij}} s_j - \bar{b_i}); \forall i \in m$$

- Parametric variance constraint corresponding to the mean value objective function,

$$\sum_{j=1}^{n} \sigma_{z_j} s_j \leq \rho \sum_{j=1}^{n} \bar{z_j} s_j \tag{3.21}$$

  The parameter ρ corresponds to the value of *coefficient of variation*, if that information is available. This information is required only in situations when the objective function itself has random parameters.

- The set of non-random constraints, and the non-negativity conditions in the original optimization problem.

3.3 Parametric Models

In this section, we would present the models corresponding to each optimization metric, delay, leakage power, dynamic power and crosstalk noise. The delay and power models have been adapted from the literature, whereas a novel crosstalk noise model has been developed as part of this work. This crosstalk model identifies noise as a linear function of the sizes of the driver gates. The dis-

cussion focuses on identifying a relationship between these metrics, and formulates the mathematical programming models required for optimization. Also, the device level and interconnect process variation are briefly visited.

3.3.1 Delay, Process Variations and Spatial Correlation

Reducing the size of a gate (say s_i), reduces the intrinsic gate capacitance of gate i, the power consumption, and the fan-in load capacitances of i. In the linear delay model [95], the delay is modeled as a function of the gate sizes, as shown in (3.22).

$$d_i = a_i - b_i s_i + c_i \sum_{j \in fo_i} s_j \tag{3.22}$$

where, d_i is the delay of gate i, s_i is the size of gate i, and s_j corresponds to the sizes of all the fan-out gates of i. The coefficients a_i, b_i, c_i are empirically determined by extensive SPICE simulations for each gate in the standard cell library for all combinations of sizes and fan-out. Specifically, b_i corresponds to the impact of channel length (L_{eff}) on the delay of a gate, and c_i corresponds to the impact of oxide thickness (t_{ox})on the delay. Thus, the delay model incorporates the impact of device process variations.

The uncertainty due to parameter variations in gate sizes is modeled according to (3.23), which is expressed in terms of nominal delay (d_i), and random parameters X_j and X_r, determining the correlated and independent variations respectively.

$$D = d_i + \sum_{j=1}^{n} d_j X_j + d_r X_r \tag{3.23}$$

Here, X_j models the principal components of correlated random variables with the corresponding d_j values evaluating the sensitivity of delay. $X_r \sim N(0,1)$ models the random component of variations in all process parameters lumped into a single term, and d_r is the standard deviation in delay due to these random variations. The magnitude of d_j and d_r is determined by extensive simulations.

In the linear delay model, only the process variation effects in the gate sizes (L_{eff} and t_{ox}) are incorporated, but not the interconnect variations. The interconnect variations in today's gigahertz designs are high, and can cause up to 25% variation in the clock skew, as shown in a paper by Liu *et al.* [68].

Also, these variations can not be incorporated in a simple nominal-worst case type of analysis. So, in this model, the effect of the interconnect variations are addressed in a mean-variance approach at the timing constraints level. The optimal delays identified through an unconstrained delay minimization as a first step in the optimization process are used as constraints for simultaneous optimization of power and crosstalk noise. We incorporate a conservative 10% variance around the mean of the best case timing values, corresponding to the interconnect variation effects.

The spatial correlations are modeled using a grid based correlation model proposed in [62]. According to this modeling, the complete design is divided into different number of regions. The gates that are in same region are highly correlated and the variation effects on all of them are similar, whereas the variation effects on the gates that are in different regions are different and are less correlated. These effects are incorporated in (3.22) to evaluate the values of b_i and c_i, approximating the variations in channel length and gate oxide thickness respectively.

3.3.2 Leakage and Dynamic Power

The power models proposed in [51] for dynamic and leakage power are adapted in our optimization formulation. The dynamic power dissipation $P_d(i)$ of a gate i in each clock cycle t_c depends upon the transition probability tp_i, the power supply voltage V_{dd}, and the load capacitance L_i. Load capacitance on a gate i is given by (3.24).

$$L_i = W_i + \sum_{k=fo_i} C_i(\bar{k}) s_k \tag{3.24}$$

where W_i is the wire capacitance, and $C_i(\bar{k})$ is the gate capacitance of the unit sized fan-out gate. The equation for dynamic power dissipation can be given as (3.25).

$$P_d(i) = tp_i \sum_{k=fo_i} C_i(\bar{k}) s_k V_{dd}^2/(2t_c) \tag{3.25}$$

The average leakage power dissipation $P_l(i)$ of gate i as a function of its size s_i, and the transition probabilities are given by (3.26).

$$P_l(i) = [\sum_r P(i,r) I_l(\bar{i},r)] s_i V_{dd} \tag{3.26}$$

where, $I_l(i,r)$ is given as the leakage current of gate i for the input pattern r, $P(i,r)$ is the leakage power for gate i corresponding to the input pattern r, $I_l(\bar{i},r)$ is the leakage current for the unit sized gate i and input pattern r, s_i is the size of gate i, and V_{dd} is power supply voltage.

From (3.25) and (3.26), it is identified that the leakage power of a gate is directly proportional to its size, and the dynamic power is proportional to the sum of the sizes of its fan-out gates.

3.3.3 Crosstalk Noise

The coupling capacitance effects in the circuits are substantial, and present a major threat to the reliability of the designs. They induce crosstalk noise on the coupled nets leading to timing yield failures. Although, the effect of crosstalk noise on a net can be reduced by using techniques like wire sizing, wire shielding, wire spacing, driver sizing (victim and aggressor), and receiver sizing (victim and aggressor), the most effective technique for reducing the crosstalk noise at the post-layout level is primarily driver sizing. In this technique, the driving gate of the net (often referred as the *victim net*), and all other driver gates of the nets that have a coupling effect on that victim net (referred as *aggressor nets*) are sized. The up-sizing of the gates increases the signal strength on corresponding net, and hence reduces the coupling effects.

Figure 3.2 shows a simple example of the noise on a net in a single victim-aggressor pair setting. Here, G1, G2, G3, and G4 are the gates sizes, C1 and C2 are the coupling capacitances between the wires, and Net1 and Net2 are the internal resistances. As discussed in [43], the major contributors to the crosstalk noise on a net are the sizes of the victim driver (G1), and aggressor driver gates (G2). If the size of the driving gate of the net is increased, the signal strength on the net increases, thereby reducing the coupling noise on the net. Also, if the size of the driver gate of the coupled net is decreased, the noise on the victim net decreases. However, such down-sizing will increase the noise on the coupled net. Similarly, the up-sizing of the victim driver will increase the noise on the coupled nets. Thus, the sizing has symmetric effects on the coupled nets, and a gate is an aggressor as well as a victim at the same time.

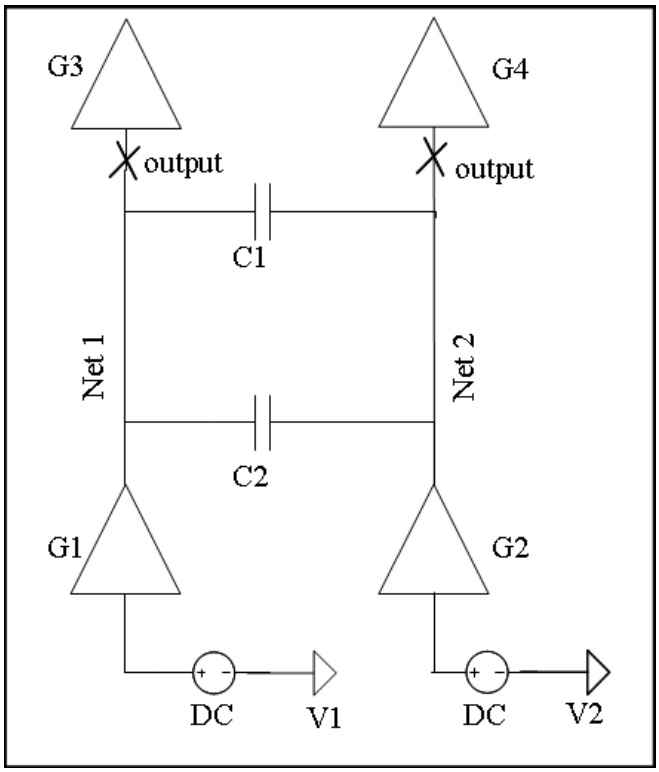


Figure 3.2 A coupling structure with single victim-aggressor pair setting. Here, G1, G2, G3, and G4 are the four gates, Net1 and Net2 are the two nets, and C1 and C2 correspond to the two half-coupling capacitances between the nets. For the victim net, Net1, G1 is the victim driver gate and G2 is the victim receiver gate. G3 is the aggressor driver gate for Net1 and G4 is the aggressor receiver gate for Net1.

The relationship between the sizes of the driving gates of the coupled nets is incorporated in formulating a simple crosstalk noise model. Here, the crosstalk noise on a net N_i is given as (3.27)

$$N_i = F_i\left(\sum_{j \in coupleds_i} (s_i - s_j)\right) \forall s_i \in n \tag{3.27}$$

where, s_i is the size of the driving gate of the net, j is the set of all the coupled nets corresponding to i, and s_j is the size of the driver gate of the coupled net in an n-gate design. Hence, for every gate, the noise on its fanout net is a function of the total cross-coupling capacitance on the net.

The effect of crosstalk noise in a circuit can also be minimized at another level by incorporating the noise margin constraints in the model. These constraints control that the maximum noise a net can

tolerate. In the present setting, the maximum tolerable noise on net j is given by (3.28).

$$U_j = H(\sum_{k=fo_j} (l_j - u_k)) \forall s_k \in coupled(s_j) \tag{3.28}$$

where, l_j and u_k correspond to the minimum and the maximum size of the gates available in the cell library.

3.4 Stochastic to Deterministic Gate Sizing

In this section, a relationship between the dynamic power, leakage power, and crosstalk noise, is first identified as a function of gate sizes, and a gate sizing approach is formulated in a mathematical programming model. These metrics are incorporated in the objective function, with the delay and noise tolerance as the constraints. The impact of process variations in different design parameters such as gate size, and interconnects is addressed in the delay constraints. Once the problem is formulated as a stochastic model, it is converted to the linear deterministic mean-variance equivalent model using the results from the methodology discussed in Section 3.2.

3.4.1 Stochastic Optimization Problem

To formulate the objective function for the multi-metric optimization problem, a relationship between the leakage power, the dynamic power, and the crosstalk noise is derived as a function of the size of gates in the design. In section 3.3, we derived these relationships independently, which can be summarized as follows:

- In Equation 3.25, it is identified that the dynamic power dissipation of a gate is primarily affected by the total size of its fan-out gates in the circuit. Thus, for a gate (say i), the total number of gates its fan-in nets are connected to determine the impact of the gate i on the dynamic power of the gates that are in its fan-in. Hence, the weight for sizing this gate i is proportional to the number of gates that it is connected to in their fan-out.

- From Equation 3.26, it is shown that the leakage power of a gate is directly proportional to the size of the gate, and hence has a direct impact. So, increasing the size of the gate would increase the leakage power dissipation of the circuit.

- The crosstalk noise on a net has an inverse relationship with the size of its driver gate. If the driver gate is sized up, the signal strength on the net increases, and hence the crosstalk noise on the net reduces. However, the up-sizing of the gate has an adverse effect on the coupled nets. By up-sizing the driver gates of the coupled nets, the noise on their corresponding output nets can be reduced.

These performance metrics are now modeled in a single objective function, which is optimized in the presence of delay constraints. The impact of these parameters on the size of the gate is incorporated by multiplying the dimensionless normalized coefficients κ, ν, and ξ, referring to the impact of a gate size on the leakage power, dynamic power, and crosstalk noise respectively. The coefficient κ is directly proportional to the size of the gate, and coefficient ν is a function of the normalized impact of the gate size on the gates that are in its fan-in. So if a gate i is in the fan-out of a large number of gates, the impact of up-sizing i will be higher for the circuit. However, ξ is inversely proportional to the size of the gate, and its normalized value is a function of the maximum coupling capacitance of its corresponding net with the aggressor nets.

The objective function for the optimization problem is given by the following equation:

$$Minimize\ GS = \sum_{i=1}^{n} (\kappa s_i + \nu s_i - \xi s_i) \tag{3.29}$$

where, s_i is the size of the gate i, and n is the total number of gates in the design.

Now, to derive the delay constraints for the multi-metric optimization problem, the deterministic best case delays for each gate in the paths are identified by performing a linear programming (LP) optimization under the path delay constraints (3.22), and the noise margin constraints (3.28). Since delay optimization is the primary objective in any circuit optimization technique, the delay is optimized as a pre-processing step. As shown in (3.30), the delay (t_{spec}) is the objective function, which is minimized to identify the best possible circuit delay. The design constraints in terms of the node delays in the paths form the constraints for the problem. Specifically, the constraints ensure that a gate $i+1$, that is connected in the fan-out of another gate i, has a delay greater than the total delay of the path till gate i and the internal gate delay of i. The nominal (mean values) case delay coefficients are used during this optimization. This deterministic optimization solution generates the delay specifications

for all the paths in the design, which are used as constraints in the next steps.

$$
\begin{aligned}
&min\ t_{spec} \\
&s.t.\ \ at_i(p) + d_i \leq at_{i+1}(p) \forall i \in n; \forall p \in P \\
&d_i = \sum_{i \in p} (a_i - \bar{b_i} s_i + \bar{c_i} \sum_{j \in fo_i} s_j) \\
&l_i \leq s_i \leq u_i \forall i \in n
\end{aligned}
\tag{3.30}
$$

where, $at_i(p)$ is the arrival time at the gate i in path p, d_i is the internal gate delay of i, and $at_{i+1}(p)$ is the arrival time at the next gate $i+1$ in the path p. The values $\bar{b_i}$, and $\bar{c_i}$ are the mean parameter coefficient values, and P is the set of all the paths in the design.

After the delays are calculated, the multi-metric optimization problem is formulated for simultaneous optimization of crosstalk noise, leakage power, and dynamic power under delay and noise margin constraints. The stochastic multi-metric optimization problem is given as (3.31).

$$
\begin{aligned}
&min\ GS = \sum_{i=1}^{n} (\kappa_i + \nu_i - \xi_i) s_i \\
&s.t\ \ d_p \leq t_{spec} \forall p \in P \\
&d_p = \sum_{i \in p} (a_i - b_i s_i + c_i \sum_{j \in fo_i} s_j) \\
&N_i \leq U_i \forall i \in n; l_i \leq s_i \leq u_i \forall i \in n
\end{aligned}
\tag{3.31}
$$

This optimization problem is stochastic in nature, since d_p contains the parametric variation coefficients b_i and c_i corresponding to the gate size variations. Also, t_{spec} is a random parameter since the variations due to interconnects are not accounted. The interconnect variations can be incorporated in the model by considering t_{spec} as a distribution rather than a number.

The next step in the problem is to convert this stochastic problem into an equivalent deterministic problem based upon the concepts of expected utility maximization and risk minimization, discussed in Section 3.2.

3.4.2 Deterministic Equivalent Model

If the objective function contains random variables, the expected utility approach can be used to transform the original stochastic optimization problem to the deterministic equivalent model, either by minimizing the variance subject to mean value constraints, or by maximizing the mean value subject to the parametric variance constraints, as discussed in the Equations (3.18– 3.21). In the gate sizing problem, this corresponds to the randomness in the parameters κ, ν and ξ. The equivalent objective function for such modeling then becomes (corresponding to Equation (3.19)),

$$min \sum_{i=1}^{n} (\bar{\kappa_i} + \bar{\nu_i} - \bar{\xi_i}) s_i \tag{3.32}$$

where $\bar{\kappa_i}$, $\bar{\nu_i}$, $\bar{\xi_i}$ correspond to the mean values of the parameters. In case of non-random κ, ν and ξ, the deterministic values will correspond to the mean values.

The constraints for the new gate sizing problem under the scarce information, and utility maximization scenario are derived as follows:

- The linear proxy constraints in terms of variance of the random parameters, corresponding to the constraints derived in Equation (3.20). Please note that in this work we have considered a 10% standard deviation in the t_{spec} value from its mean value identified during the first step of the optimization process. The approximation is more conservative than the one derived in [68]. A more accurate estimate can be incorporated without any modifications in the model.

$$\sigma_{d_p} + \sigma_{t_{spec}} < \sqrt{3(1-\omega)}(\overline{d_p} - \overline{t_{spec}}); \forall p \in P \tag{3.33}$$
$$\sigma_{d_p} \equiv \sum_{i \in p} (\sigma_{a_i} - \sigma_{b_i} + \sigma_{c_i} \sum_{j \in fo_i} s_i)$$
$$\overline{d_p} \equiv \sum_{i \in p} (\overline{a_i} - \overline{b_i} + \overline{c_i} \sum_{j \in fo_i} s_i)$$

  The value of the utility maximization parameter ω can be experimentally determined, and can be kept at a high level of 0.9, 0.92 or even 0.95.

- The parametric variance constraint corresponding to the mean valued objective function is given by,

$$\sum_{i=1}^{n} (\sigma_{\kappa_i} + \sigma_{\nu_i} - \sigma_{\xi_i}) s_i \leq \rho \sum_{i=1}^{n} (\overline{\kappa_i} + \overline{\nu_i} - \overline{\xi_i}) s_i \tag{3.34}$$

where ρ is the coefficient of variation. This constraint is required only in situations where the objective function also has random parameters (say random cost parameters) in it. However, in this work, the modeling does not consider the randomness in the objective function parameters, and hence this constraint is not utilized at all. If the coefficients in the objective function are considered as random during the modeling, then experimentation with different values of ρ provides a complete frontier of the optimal solutions.

- The Non-random constraints, and the conditions corresponding to the available gate size ranges are maintained.

$$N_i \leq U_i; \forall i \in n \tag{3.35}$$

$$l_i \leq s_i \leq u_i; \forall i \in n \tag{3.36}$$

Thus, the stochastic problem defined in Equation (3.31), is converted into a deterministic equivalent multi-metric model given by Equations (3.32), (3.33), (3.34), (3.35) and (3.36) which can be solved using any linear programming optimization tool.

3.5 Experimental Results

In this section, we present the simulation results to verify the efficiency, accuracy and efficacy of this methodology. First, a sensitivity analysis is performed to evaluate the sensitivity of the expected utility assurance constant (ω) with reference to the timing yield and the optimization of the metrics. The analysis helps in identifying the optimum value of ω corresponding to the optimization requirements. The risk averse optimization algorithm is then evaluated on the benchmark circuits for optimization of different metrics and the execution time. Also, the approach is compared with a recently proposed device variation aware mathematical programming based approach. The algorithm is then used to perform single metric optimization, and the results are compared with the multi-metric optimization values to evaluate the relative impact of a single metric optimization on the sub-optimality introduced in other metrics. Finally, the impact of incorporating interconnect variations in addition to the gate size variations in the algorithm is analyzed. It is important to note that this method has been transformed from a path based approach to an equivalent node based approach according to the technique proposed in [47]. This controls the size of the problem to further improve the runtime and

feasibility for large circuits. The sub-optimality introduced due to the transformation is close to 2% for circuits with 20 levels of logic.

3.5.1 Setup

The multi-metric optimization algorithm for gate sizing under process variations was rigorously tested on the ITC'99 benchmark circuits. The setup consisted of three steps, as listed below.

- The RTL level VHDL net-lists of the benchmark circuits were extracted for generating gate level Verilog files using the *Synopsys Design Compiler* tool. These gate level Verilog files and the TSMC 180nm Standard Cell libraries (LEF, TLF, DB Files etc.) were then used to place and route the designs and generate the DEF files, cell delay information etc. using the *Cadence Design Encounter* toolkit. The benchmark circuits were synthesized using TSMC 180nm libraries since the lower level libraries were not available to us.

- The parasitic resistance and capacitance information (SPEF file) was extracted from the routed designs using the Cadence *Fire N' Ice* RC extractor. This information was utilized for extracting the coupling capacitance from the routed circuits. A PERL script was written to extract the coupling capacitance information of each net with its top three coupled nets from the SPEF file.

- The delay coefficients for available gate sizes (1x - 6x) and fan-outs of the standard cells in the TSMC 180nm standard cell library were characterized using the HSPICE simulations. Also, the variations in gate sizing parameters were assumed to be 25% of the nominal values, which were appropriately translated to the coefficients a, b and c in Equation 3.22. A conservative estimate of 10% variance for the interconnect variations in the best case delay constraints was incorporated in the model. The best case delays were identified in a pre-processing step, through the unconstrained linear programming optimization.

After the delays, coupling noise etc. were calculated for the benchmark circuits, the stochastic gate sizing optimization problem was formulated. The algorithm for formulating the stochastic linear programming model was programmed in C language. Next, the stochastic problem was converted into a deterministic linear programming equivalent model through the expected utility maximization approach. The linear program was then converted to the standard AMPL format, which was solved using

the robust KNITRO [96] optimization solver. KNITRO uses interior point and active set methods for optimization and is capable of utilizing multiple processors. This solver is specifically designed to solve problems with large dimensionality. A detailed system flow is shown in Figure 3.3.

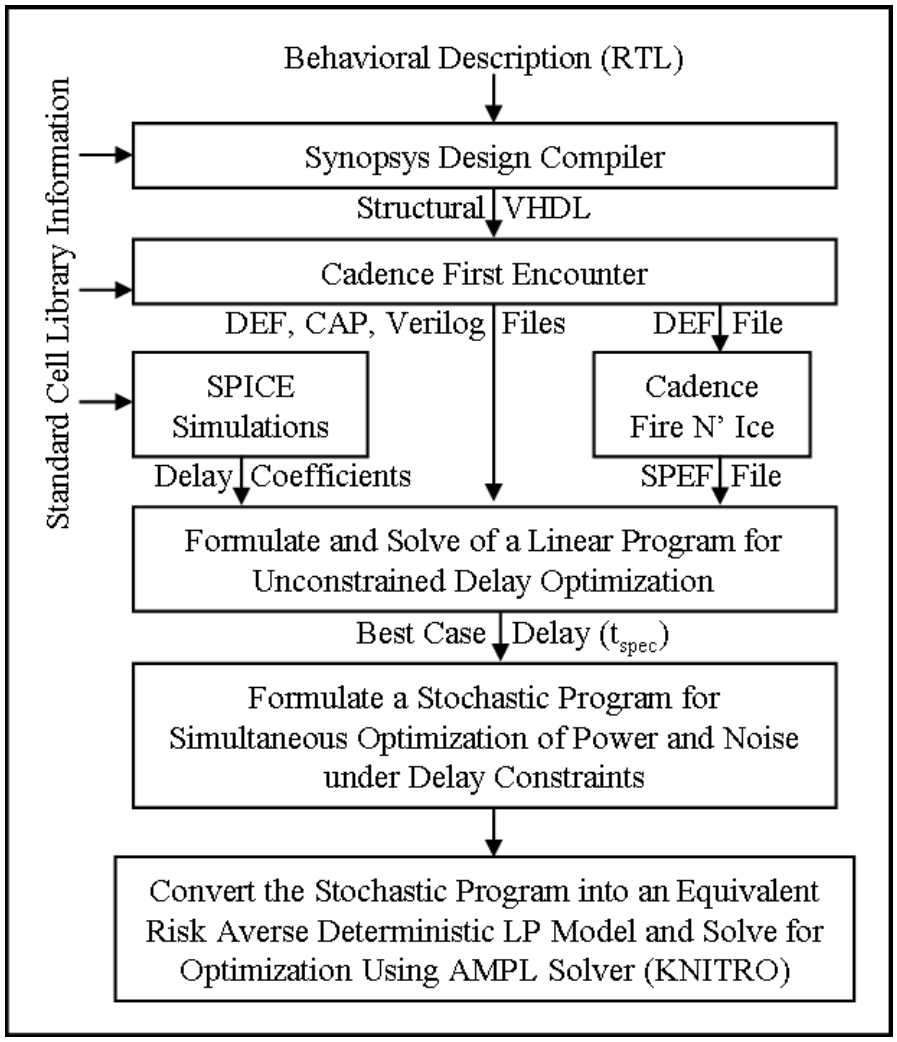


Figure 3.3 Simulation setup for the risk averse gate sizing optimization problem. The Synopsys and Cadence tools are used to generate DEF, CAP, Verilog and SPEF files for each benchmark circuit. These files, and the delay coefficients (a, b, c)determined by the extensive SPICE siimulations are utilized in formulating the unconstrained linear programming problem, which is solved to evaluate the best delays for each benchmark. The multi-metric stochastic optimization problem is then formulated and converted to the equivalent risk averse deterministic problem. These linear programming problems are then solved using the KNITRO optimization solver.

3.5.2 Sensitivity of Utility Assurance Constant

The expected utility constrained optimization process involves identification of appropriate value for the parameter ω, which determines the utility of satisfying each constraint in the optimization problem. In the domain of circuit optimization, each constraint is critical, and hence the satisfaction of the constraints is central to the optimization process. Thus, the greatest bearable shortfall is kept as zero. The expected utility assurance constant ω evaluates the utility of each constraint in the model. Intuitively, a high value of ω would ensure that each constraint in the problem is critical, and thus needs to be satisfied. However, such a high value of ω may result in the under-achievement of the optimization goals, whereas a low value would result in unreliability of the optimization process and consequently the yield. To evaluate the consistency of this intuition, and to identify the optimum range of values for the utility assurance constant, experiments were performed on the benchmark circuits to determine the average change in timing yield and the metric optimization values.

The timing yield of a circuit determines the probability that the circuit satisfies timing constraints. In this model, the best timing specification (t_{spec}) is identified during the unconstrained delay optimization, and is used as a constraint during the multi-metric optimization. In the risk averse mathematical programming formulation with delay constraints corresponding to each node, a high timing yield is obtained since each timing constraint is satisfied. However, a timing value of the critical gates closer to the t_{spec} value is at *occam's razor*, with a high probability of failure due to process variations effects. The timing yield of the circuit is higher if the difference between the maximum delay of the circuit after optimization and the timing specification (t_{spec}) is large. Thus we evaluate the improvement in the timing yield of the benchmark circuits for various values of ω as $(t_{spec} - t_{max}) * 100/t_{max}$, where t_{max} is the maximum delay of the circuit after the multi-metric optimization is performed. The graph shown in figure 3.4 displays the timing yield improvement values for the benchmark circuits. The results endorse the intuition that as the ω value decreases, the timing yield of the circuit decreases, since the satisfaction of each constraint does not have a high utility value, and thus may not satisfy, which is the case for some designs (b11, b12, and b20) at low utility values.

The effect of ω on the objective function optimization follows a counter trend. As shown in the Figure 3.5, the objective function optimization values for different utility assurance constants (ω) as compared to the values corresponding to $\omega = 0.99$ consistently improve for each benchmark circuit.

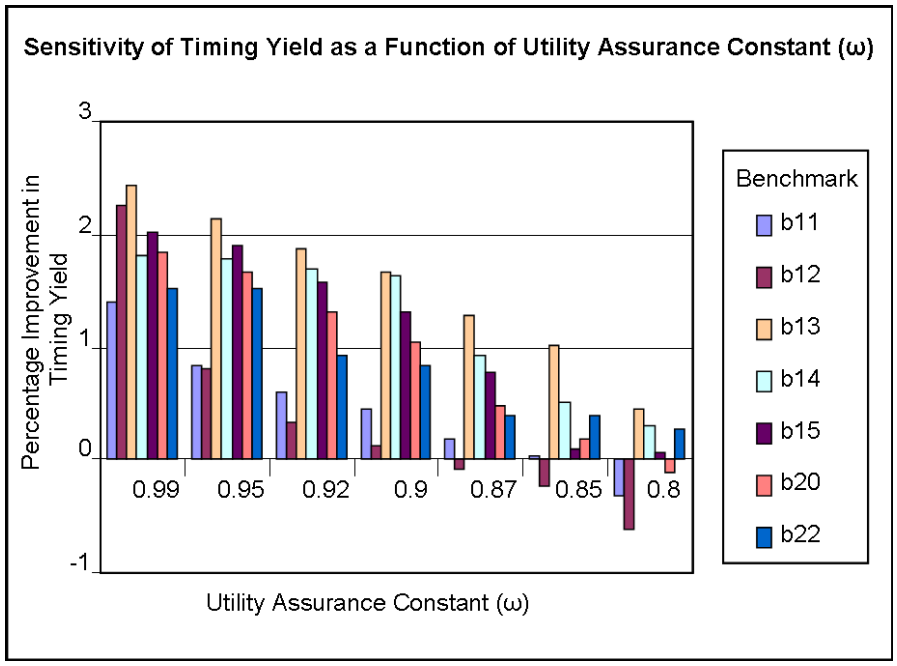


Figure 3.4 Improvement in the timing yield of the circuits for different values of ω. The timing yield improvement is given as the percentage increase in the difference between t_{spec} and the maximum delay (t_{max}) of the circuit after the multi-metric optimization is performed.

This also follows the intuition, since the utility of each delay constraint for a lower value of ω is less, there is greater margin for optimization at the cost of dissatisfaction of the constraint. An important step in this risk averse optimization process is to identify the optimum ω values based upon the optimization requirements. A good option in the general circuit optimization domain is to use a ω value that provides at-least 98-99% timing yield, as well as identifies good solution points. In this work, we have performed most of the analysis with the values of $\omega = 0.95, 0.92$, and 0.90.

3.5.3 Risk Averse Optimization Results

To evaluate the optimization values for dynamic power, leakage power, and crosstalk noise in an equally weighted multi-metric optimization setting of this risk averse gate sizing (RAGS) methodology, a comparative analysis with an existing multi-metric fuzzy mathematical programming methodology (FMP) [97, 98] has been performed. It is important to note that since the fuzzy mathematical programming method only incorporates the effect of process variations due to gate sizes (constants a_i, b_i, and c_i), the risk averse gate sizing method was also implemented with only device variations for a fair comparison. The fuzzy mathematical programming approach is a three step process that

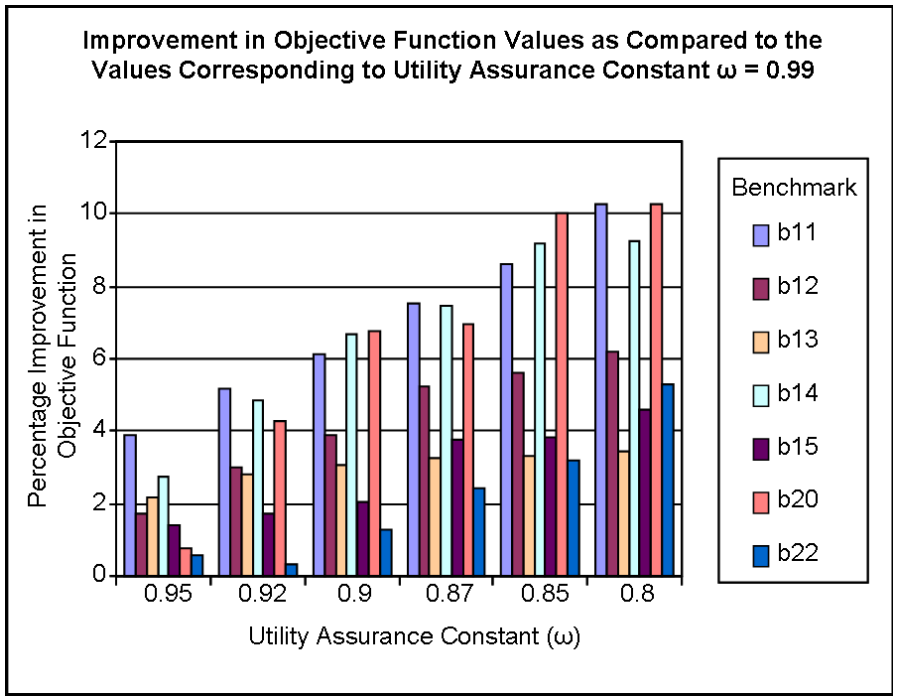


Figure 3.5 Percentage improvement in the optimization of the objective function for various values of ω. The objective function improvements are compared with the values obtained for $\omega = 0.99$.

first calculates the worst case and nominal case gate size values, with the optimal delays incorporated as constraints in a similar fashion as this approach. These values are then used to formulate a fuzzy non-linear program that is solved for optimization. The results for two values of utility assurance constant $\omega = 0.90$ and $\omega = 0.95$ are shown in the Table 3.1. The results indicate that the optimization values obtained using the risk averse gate sizing method is comparable to the fuzzy mathematical programming results. On average, the improvement in metric optimization values for each metric is approximately equal to the FMP counter-parts for $\omega = 0.95$. The RAGS performed slightly better than FMP in terms of metrics optimizations for $\omega = 0.90$. However, an important aspect of the risk averse gate sizing approach is the execution time of the algorithm. As shown in last three columns of Table 3.1, the algorithm execution time of our methodology is significantly less as compared to the FMP method. The RAGS is 5.85 times faster than the FMP method for $\omega = 0.90$ and more than 6.4 times faster for $\omega = 0.95$. This is attributed to the fact that the RAGS method is a single step linear programming method as compared to FMP method which requires three steps, and the last step is a non-linear program. This is significantly important for larger circuits like b17 which have more than 21000 gates.

This algorithm can be utilized to perform single metric optimization of the metrics depending upon the design requirements. The metric to be optimized can be prioritized by assigning a high weight vector to it. For example, if the designer intends to optimize only leakage power, then the weight vector corresponding to κ is assigned as 1 where as the other metrics ξ and ν in the objective function are assigned as 0 and 0 respectively. However, during such optimization the improvement in the optimization values of the corresponding metric comes at the cost of introducing sub-optimality in the values of other metrics. We have compared the results of single metric optimizations for dynamic power, leakage power and crosstalk noise, with the equally weighted simultaneous multi-metric optimization of all three metrics. The results of the average change in the optimization values for the metrics as compared to multi-metric optimization in all three cases is shown in Table 3.2.

The dynamic power dissipation for single metric optimization is lower than the multi-metric optimization (as shown in column 2 of Table 3.2). However, it is interesting to note that on average, dynamic power dissipation is reduced at the cost of leakage power and not crosstalk noise. This trend occurs due to the fact that during the dynamic power minimization, fewer gates are resized from the sub-optimal sizes (after delay optimization) as compared to leakage power optimization. This results in a decrease in dynamic power, but the leakage power is largely unaffected. When only leakage power is optimized, the optimization introduces sub-optimality primarily in crosstalk noise metric. This is intuitive, since the leakage power is directly proportional to the gate sizes, where-as the crosstalk noise has an inverse relationship with the gate sizes. When the single metric optimization for crosstalk noise is performed, the gate sizing problem translates into a maximization problem. The reduction in crosstalk noise as compared to the equally weighted multi-metric optimization is notable (almost 47%). This is due to the fact that the maximization problem satisfies the delay constraints much easily as compared to the minimization problem. Increasing the gate sizes reduces the crosstalk noise, as well as the gate delays. However, this increases the power dissipation of the design by 30%.

3.5.4 Optimization Considering Device and Interconnect Variations

In this set of experiments, both device level (gate size) variations as well as interconnect variations are incorporated in the model, and the impact of interconnect variations on the optimization of objective function and the timing yield is analyzed. The results are compared to the scenario where

multi-metric optimization is performed considering only gate size variations. The overall variations due to interconnects are assumed to be 10% of the nominal case values, and are incorporated in the delay constraints by adding a variance of 10% to the optimal delay values (t_{spec}). The utility assurance constant ω is kept as 0.92. As shown in Figure 3.6, the risk averse gate sizing methodology ensures that the timing yield of the circuit is not sacrificed, even though the optimization results are affected. It is interesting to note that the in the presence of interconnect variations, the timing yield for most cases actually improves, since the methodology is risk averse, and intends to satisfy the constraints with high utility. Due to this, the optimization is adversely affected.

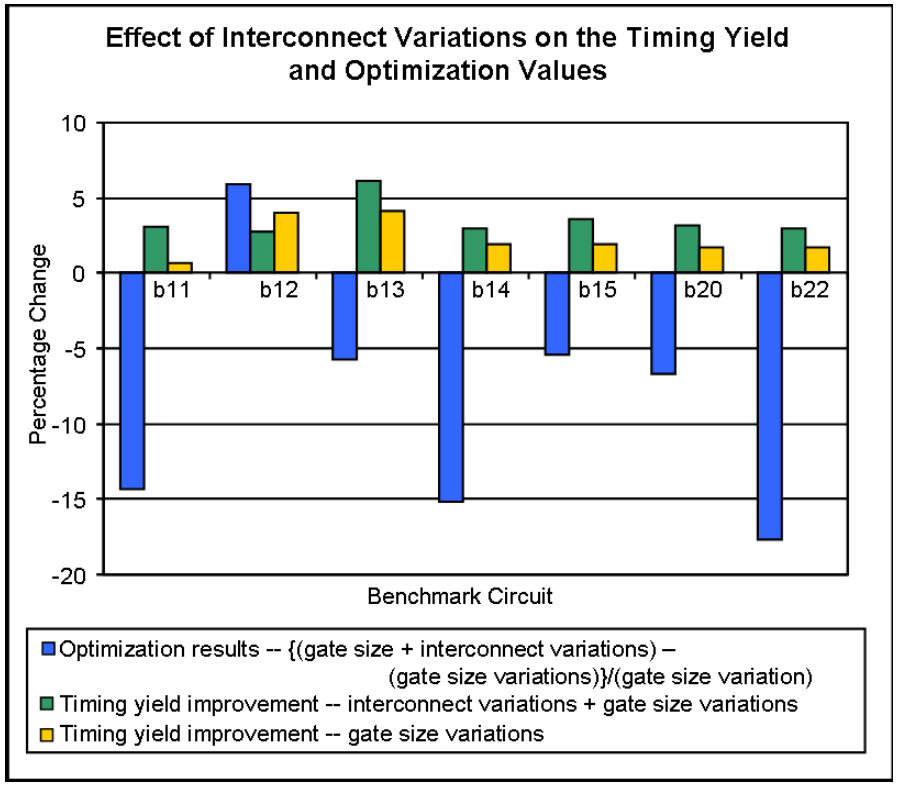


Figure 3.6 Impact of interconnect variations and gate size variations on the optimization of the metrics. In these experiments, the interconnect variations (of 10% variation from the mean value) are incorporate in addition to the gate size variations. The percentage change in the yield improvement for the two cases (gate size variations and gate size and interconnect variations respectively), and the corresponding percentage change in the objective function achievement for the latter case (gate size and interconnect variations) as compared to the former (only gate size variations) are plotted in the graph. The value for expected utility assurance is kept at $\omega = 0.92$.

3.6 Discussion

In this work, a new algorithm for simultaneous multi-metric optimization of delay, dynamic power, leakage power and crosstalk noise in presence of process variations and scarce information has been developed. The algorithm is independent of the underlying variation distribution, and can handle the impact of variations at several levels including variations due to gate sizes and due to interconnects. This expected utility maximization based methodology models the problem in a mean-variance based deterministic linear programming model, which optimizes the objectives while ensuring high levels of expected utility for constraints satisfaction. The experiments conducted on the ITC'99 benchmarks suggest that the algorithm is multi-fold faster than the existing mathematical programming algorithms available in the literature, and ensures comparable optimization results. Good optimization results and timing yields are obtained when the utility assurance constant ω is kept at the levels between 0.95 and 0.90. A comparative study between the single metric and the multi-metric optimization reveals that the improvements in a single metric are largely achieved by incorporating sub-optimality in other metrics. This method is capable of incorporating more optimization metrics like security and reliability etc., if the metric can be expressed as a function of gate sizes. In general, the methodology developed in this work is a fast, accurate, and efficient tool for nano-level post-layout gate sizing optimization for complex circuits, where the complete variation distribution information is unavailable.

Table 3.1 Comparison between equally weighted multi-metric optimization of leakage power, dynamic power and crosstalk noise for risk aware gate sizing (RAGS) and fuzzy mathematical programming (FMP).

Comparison for Expected Utility Assurance Constant $\omega = 0.90$								
ITC'99 Benchmark	Number of gates	Optimal Delay (ns)	†Performance of RAGS as Compared to FMP			Exec. Time(ET) (secs)		Speed-up of RAGS
			Dynamic Power	Leakage Power	Crosstalk Noise	RAGS	FMP	
b11	385	0.71	15.65%	18.75%	-5.91%	0.53	2.35	4.43x
b12	834	0.36	5.86%	7.52%	9.44%	9.63	38.65	4.01x
b13	249	0.26	-3.95%	-6.05%	-2.69%	0.143	0.848	5.93x
b14	4232	2.5	-5.42%	-5.59%	9.31%	23	177	7.7x
b15	4585	3.43	-1.58%	-1.26%	5.98%	54	213	3.94x
b20	8900	3.59	8.89%	9.75%	-4.58%	97	713	7.35x
b22	12128	2.63	3.69%	4.08%	1.48%	145	978	6.74x
b17	21191	2.68	3.89%	1.79%	-3.99%	349	2338	6.7x
Percentage Change			3.38%	3.62%	1.13%			5.85x
Comparison for Expected Utility Assurance Constant $\omega = 0.95$								
b11	385	0.71	13.40%	16.28%	-7.49%	0.51	2.35	4.6x
b12	834	0.36	4.03%	5.33%	14.56%	8.38	38.65	4.61x
b13	249	0.26	-4.82%	-7.59%	2.17%	0.131	0.848	6.47x
b14	4232	2.5	4.66%	4.95%	-9.20%	19	177	9.31x
b15	4585	3.43	-6.78%	-6.46%	13.21%	49	213	4.34x
b20	8900	3.59	-9.12%	-9.36%	0.82%	88	713	8.1x
b22	12128	2.63	1.50%	1.75%	-4.66%	141	978	6.93x
b17	21191	2.68	2.53%	1.21%	-2.34%	330	2338	7.08x
Percentage Change			0.68%	0.77%	0.88%			6.43x
†: The Percentage Change in the optimization of each metric is computed using the formula $(ET_{FMP} - ET_{RAGS}) * 100/ET_{RAGS}$								

Table 3.2 Comparison between equally weighted multi-metric optimization with $\omega = 0.92$ and single metric optimization for dynamic power, leakage power, and crosstalk noise metrics.

ITC'99	†Dynamic Power Optimization			‡Leakage Power Optimization			∓ Crosstalk Noise Optimization		
Benchmark	*DP	**LP	Noise	DP	LP	Noise	DP	LP	Noise
b11	33.05%	-7.21%	-9.57%	-5.90%	36.94%	-22.37%	-11.73%	-13.09%	29.49%
b12	35.87%	-9.03%	-20.79%	-8.29%	37.36%	-33.76%	-11.20%	-12.09%	28.28%
b13	1.18%	-15.54%	3.01%	-13.09%	1.83%	12.29%	-18.51%	-18.99%	77.61%
b14	17.90%	-3.54%	1.98%	-2.87%	19.25%	-26.84%	-7.54%	-7.77%	16.57%
b15	40.47%	-8.67%	-2.82%	-7.97%	41.20%	-21.88%	-21.37%	-21.30%	42.91%
b20	9.15%	-14.17%	13.76%	-13.15%	10.07%	-29.70%	-18.03%	-18.75%	76.01%
b22	18.07%	-11.49%	6.45%	-10.83%	19.33%	-19.25%	-13.24%	-13.84%	57.59%
Average	22.24%	-9.95%	-1.12%	-8.87%	23.71%	-20.21%	-14.52%	-15.12%	46.92%

†: Percentage Change in metric values when single metric optimization for Dynamic Power is performed
‡: Percentage Change in metric values when single metric optimization for Leakage Power is performed
∓: Percentage Change in metric values when single metric optimization for Crosstalk Noise is performed
*: Dynamic Power **: Leakage Power

CHAPTER 4

INTEGRATED FRAMEWORK FOR CIRCUIT OPTIMIZATION

In the nanometer regime, the transition of the process technology from one generation to the next is contributing toward the identification of new metrics that can significantly affect the performance and reliability of the designs. Thus, circuit optimization techniques developed to address the current generation VLSI optimization issues may not be applicable to the future generation optimization requirements. Also, a wide spectrum of devices incorporate VLSI circuits as an integral part of the design. The design requirements of these devices vary widely. For example, mobile devices primarily require low power dissipating design, while mission critical devices must ensure that the design is reliable. Thus, there is a need for new horizontally and vertically integrated circuit optimization solutions that are completely reconfigurable in terms of the metrics to be optimized, the optimization methodology to be utilized, and the relative priorities with which the metrics are optimized. Thus, in this chapter, a framework to optimize multiple performance metrics in a unified manner is developed. In this variation aware optimization model, a relationship between the optimization metrics (like dynamic power, leakage power, and crosstalk noise) as a function of gate sizes is incorporated in the objective function. The delay values obtained from unconstrained delay optimization, and noise margins obtained from the coupling capacitance information form the constraints form the optimization problem, which is then solved for simultaneous optimization of multiple metrics. The framework is independent of the optimization methodology, and can be implemented using any mathematical programming approach. It is completely reconfigurable and generalized such that metrics can be selected, removed, or prioritized for relative importance depending upon the design requirements. This framework is implemented, and tested on ITC'99 benchmarks for different combinations of multi-metric and single metric optimizations of delay, dynamic power, leakage power, and crosstalk noise. The results indicate that the approach identifies good solution points, and is an efficient mechanism for post-layout optimization via gate sizing.

4.1 Need for Integrated Framework

In the nano-meter regime, the increase in density and complexity of the VLSI circuits has affected the circuit optimization process in several ways. First, the inter-relationships between the metrics like delay, power, and crosstalk noise have become more intricate in such a manner that the optimization of one metric may worsen the optimality of other metrics. Thus, the optimization of a single metric may no longer be adequate. Second, due to aggressive scaling, the wiring density and consequently the aspect ratios in metal lines have increased, there-by magnifying the impact of coupling capacitance between the nets. The crosstalk noise induced between the coupled nets could cause functional failures in the circuits. As a simple example, if a gate sizing technique is aimed at only power minimization of a given circuit, based on timing constraints, the resulting gate size configuration could potentially have a high interconnect crosstalk noise.

According to the technology trends [99], these effects and uncertainties are expected to worsen in future, and optimization methodologies which result in a performance shift from one objective to another will not be acceptable. From the VLSI circuit optimization perspective, an important challenge is to identify vertically as well as horizontally integrated solution methodologies [14]. This necessitates the examination of new approaches that can simultaneously optimize multiple design parameters for feasible solutions to circuit design problems.

Another important aspect in nano-level VLSI design and the optimization process is to address the effect of process variations, which introduce uncertainty in the geometries of devices like gate sizes (gate length, oxide thickness etc.) of the fabricated circuits. As shown in Figure 2.5, at sub-100nm levels, the intra-die parametric and systematic variations are comparable to the random variations [1]. The effects of layout schematics as well as parametric variations increase significantly due to the shrinking geometries. The uncertainty due to these manufacturing variations impacts the performance characteristics and the reliability of the circuits. An optimization model that does not incorporate the impact of process variations could result in inaccurate analysis.

In this work, we present a new variation aware multi-metric gate sizing framework that can be used to perform optimization of several metrics like delay, leakage power, dynamic power, and crosstalk noise etc. This approach is completely reconfigurable and generalized in terms of its capability to incorporate new metrics and selectively prioritize the metrics depending upon the design requirements,

with minimal changes in the model. More importantly, any mathematical programming approach can be utilized within this framework, to solve the optimization problem. An important aspect of this approach is the identification of the inter-relationships between dynamic power, leakage power, and crosstalk noise in terms of gate sizes, and modeling them in a unified framework.

In this framework, since delay is the primary objective in any circuit optimization process, it is optimized with highest priority as a first step in the process. The delay values obtained from unconstrained delay optimization are then used as constraints during the simultaneous optimization of other metrics: dynamic power, leakage power and crosstalk noise. The process variation effects due to gate sizes (channel length, oxide thickness) are incorporated in the model, and a grid based model is used to address the spatial correlation effects [62].

The state of the art research in VLSI design optimization that consider the impacts of process variations has been discussed in details in the Section 2.4 of Chapter 3. The methods implemented for optimizing various metrics has been studied and compared. Also, the different mathematical programming techniques available in the literature, their properties and limitations are reviewed in that section. One of these, or any other mathematical programming methodology may be used as the optimization tool in this framework, without any apparent modification in the modeling. The readers might want to visit Sections 2.4 and 3.3 again to follow the material presented in the rest of this chapter.

The rest of the chapter is organized as follows. In Section 4.2, single metric mathematical programming optimization models for leakage power, dynamic power, and crosstalk noise are derived on the basis of the parametric models derived in the Section 3.3 of Chapter 3. The relationship between the design parameters in terms of gate sizes, and the steps involved in modeling the problem in a multi-metric optimization framework are presented in section 4.3. Also, a mathematical programming approach that is utilized for optimization in this work is briefly discussed. In Section 4.4, the simulation setup, experimental results for various optimization scenarios, and analysis are presented, followed by a summary and discussion in Section 4.5.

4.2 Single Metric Optimization Models

An important step in modeling any problem in a mathematical programming framework is to identify the relationships between the design parameters. A model that relates the parameters in a

simple yet accurate fashion improves the optimization process both in terms of the efficiency and the applicability of the model. In this section, we present the general mathematical programming formulations for optimization of each metric, delay, power, and crosstalk noise.

4.2.1 Unconstrained Delay Optimization

Delay is an important optimization metric in any circuit optimization problem. Since a delay optimized circuit has higher timing yield, the delay is considered as the most critical metric to be optimized in any generalized framework. In the presence of process variations at the nanometer level, delay uncertainty can be reduced by performing unconstrained delay optimization. The mathematical programming model for unconstrained delay optimization is given by equation (4.1). The noise margin constraints that control the maximum noise a net can tolerate has been derived in Equation (3.28). The design constraints in terms of the node delays in the paths form the constraints for the problem.

$$
\begin{aligned}
& min\ t_{spec} \qquad (4.1)\\
& s.t\ \ at_i(p) + d_i \leq at_{i+1}(p) \forall i \in n; \forall p \in P\\
& N_i \leq U_i \forall i \in n\\
& d_i = \sum_{i \in p} (a_i - b_i s_i + c_i \sum_{j \in fo_i} s_j)\\
& l_i \leq s_i \leq u_i \forall i \in n
\end{aligned}
$$

Similar to the Equation (3.30), the $at_i(p)$ is the arrival time at the gate i in path p, d_i is the internal gate delay of i, and $at_{i+1}(p)$ is the arrival time at the next gate $i+1$ in the path p. U_i is the upper bound on the noise margin, and N_i is the noise margin of the current net. The values b_i, and c_i are the uncertain parameter coefficient values, and P is the set of all the paths in the circuit. The linear delay model is adapted from [95] and has been described in Equation (3.22).

4.2.2 Power Optimization Under Delay Constraints

The power models proposed by Gao and Hayes [51] have been adapted to identify the leakage power and the dynamic power as a function of gate sizes in Equations (3.26) and (3.25) respectively.

These equations can be used to formulate the mathematical programming model for power optimization under delay constraints.

From (3.26) and (3.25), it is identified that the leakage power of a gate is directly proportional to its size, and the dynamic power is proportional to the sum of the sizes of its fan-out gates. Hence, using the linear delay model, the problem of leakage and dynamic power optimization under delay constraints can be given by (4.2).

$$\begin{aligned} & min \sum_{i=1}^{n} (\theta * s_i + \phi * s_i) \\ & s.t.\ d_i \leq d_i(max) \forall i \in n \\ & l_i \leq s_i \leq u_i \forall i \in n \end{aligned} \tag{4.2}$$

where, θ corresponds to the normalized impact (weight) of gate size on the leakage power, and ϕ corresponds to the normalized impact of the gate size on the fan-in gates of the design, and effectively the dynamic power impact. Here, d_i is the delay of gate i, $d_i(max)$ is the upper bound on the delay of gate i, according to the timing specifications in a node based model, n is total number of gates in the design, and l_i and u_i are the minimum and maximum available gate sizes in the standard library, respectively.

4.2.3 Crosstalk Noise Optimization Under Delay Constraints

The relationship between the sizes of the driving gates of the coupled nets, derived in Equation (3.27) can be incorporated in formulating a linear programming formulation for the crosstalk noise optimization under delay constraints. Here, for every gate s_i, the noise on its fan-out net is a function of the total cross-coupling capacitance on the net. So, given a weight vector ξ as a function of the cross-coupling capacitance on the net, such that $\xi_i \forall i \in n$, and $\xi_i \geq 0$, an equivalent node based linear programming model for crosstalk noise optimization by gate sizing can be formulated for maximizing the weighted sum of the gate sizes, under delay and noise margin constraints. The linear programming

formulation for crosstalk noise optimization can be formally stated as (4.3).

$$\begin{aligned} & max \sum_{i=1}^{n} \xi_i s_i \\ & s.t.\ d_i \leq d_{max} \forall i \in n \\ & N_i \leq U_i \forall i \in n \\ & l_i \leq s_i \leq u_i \forall i \in n \end{aligned} \tag{4.3}$$

The problem is formulated as a maximization problem to minimize the noise on each gate by maximizing its size, weighted by the impact of each gate on the crosstalk of each output net. Since, the impact of sizing is symmetric on the coupled nets, the optimal sizes of the gates are obtained that would reduce the coupling effect.

4.3 Integrated Framework for Variation Aware Gate Sizing

In this section, a detailed description of the unified gate sizing approach is presented. First, we discuss the various aspects of the objective function modeling, in which the relationships between the three metrics, dynamic power, leakage power, and crosstalk noise are captured as a function of gate sizes. The objective function should be reconfigurable in the sense that any metric can be inserted or deleted, or weighted as required. Next, the integrated framework is discussed, followed by the mathematical programming formulation for variation aware optimization. Finally, we will briefly discuss the process of converting a path based approach to a node based approach, which improves the runtime of the algorithm.

4.3.1 Objective Function Modeling

In the context of gate sizing, the impact of the three design parameters, leakage power, dynamic power, and crosstalk noise as a function of gate sizes is incorporated in the objective function that is to be optimized. Specifically, the inter-relationship of the three metrics is as follows.

- As shown in Equation (3.26), the leakage power of a gate is directly proportional to the size of the gate, and hence has a direct impact. So, increasing the size of the gate would increase the

leakage power dissipation of the circuit. The components like the input transition probabilities, leakage current and the input patterns also impact the leakage power dissipation of the circuit.

- The relationship between the dynamic power dissipation and the gate sizes is shown in Equation (3.25). The dynamic power dissipation of a gate is primarily affected by the total size of its fan-out gates in the circuit. Thus, for a gate *i*, the total number of gates its fan-in nets are connected to, determine the impact of the gate *i* on the dynamic power of the gates that are in its fan-in. Hence, the weight for sizing this gate *i* is proportional to the number of gates that it is connected to in their fan-out.

- As discussed in the previous section, the crosstalk noise on a net primarily depends upon the size of its driver gate and the sizes of the driver gates of the coupled nets. Hence, crosstalk noise has an inverse relationship with the gate size. If the driving gate of a net is sized up, the signal strength on the net increases and hence the crosstalk noise on the net reduces. However, the up-sizing of the gate has an adverse effect on the coupled nets. By up-sizing the driver gates of the coupled nets, the noise on those nets can be reduced.

These performance metrics can be modeled in a single objective function, which is optimized in the presence of delay constraints. Hence, three dimensionless normalized coefficients θ, ϕ, and ξ, referring to the impact of gate sizing on the leakage power, dynamic power, and crosstalk noise respectively are incorporated in the objective function. The coefficient θ is directly proportional to the size of the gate, and coefficient ϕ is a function of the normalized impact of the gate size on the gates that are in its fan-in. So if a gate *i* is in the fan-out of a large number of gates, the impact of up-sizing *i* will be higher for the circuit. However, ξ is inversely proportional to the size of the gate, and its normalized value is a function of the maximum coupling capacitance of its corresponding net with the aggressor nets. The higher the cross-coupling capacitance value higher is the coefficient value.

To incorporate the capability to select among the objective functions, three constants α, β, and γ are multiplied to the coefficients θ, ϕ, and ξ, controlling the impact of these coefficients on the final objective achievement. For example, if all the three objectives leakage power, dynamic power, and crosstalk noise are equally weighted, then $\alpha = \beta = \gamma = 0.33$.

The objective function is given by the following equation:

$$min \sum_{i=1}^{n} (\alpha * \theta * s_i + \beta * \phi * s_i - \gamma * \xi * s_i) \tag{4.4}$$

where, s_i is the size of the gate i, and n is the total number of gates in the design.

4.3.2 Integrated Framework

After the objective function is identified as shown in (4.4), an integrated framework for the multi-metric optimization is formulated. Figure 4.1 shows the top level flowchart for the framework. During the first step, a linear program is formulated, as shown in (4.1), with the delay (t_{spec}) as the objective function, which is minimized to identify the best possible circuit delay. The nominal (best) case delay coefficients are used during this optimization. This deterministic optimization solution generates the delay specifications for all the paths in the design, which are used as constraints in the next steps.

In the next step, any mathematical programming methodology can be implemented to perform the stochastic optimization by incorporating the delay constraints (3.22) and the noise margin constraints (3.28). The weighted coefficients θ, ϕ, and ξ are incorporated in the objective function, which allow to incorporate the priorities for optimizing the metrics according to the designer requirements. The mathematical programming formulation is given in (4.5).

$$\begin{aligned} min \sum_{i=1}^{n} (\alpha\theta + \beta\phi - \gamma\xi) s_i \\ s.t \;\; d_p \leq t_{spec} \forall p \in P \\ N_i \leq U_i \forall i \in n \\ l_i \leq s_i \leq u_i \forall i \in n \\ d_i = a_i - \bar{b_i} s_i + \bar{c_i} \sum_{j \in fo(i)} s_i \end{aligned} \tag{4.5}$$

After the problem is formulated in the mathematical programming framework, it can be converted into a standard optimization language format (AMPL etc.), and can solved using any linear programming solver.

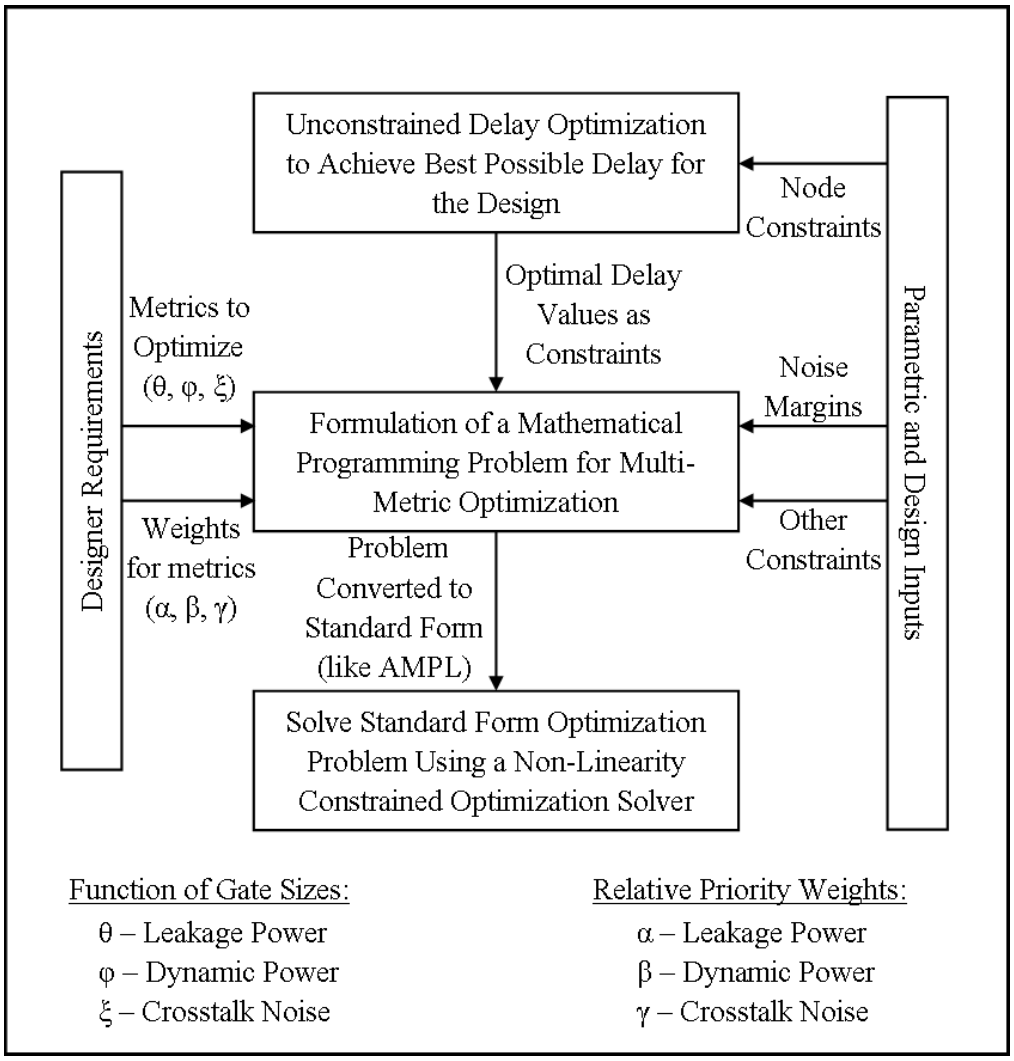


Figure 4.1 Gate sizing framework for multi-metric circuit optimization. Since delay is the primary objective in VLSI optimization, it is optimized separately as the first step in the process. Next, the other optimization metrics are simultaneously optimized by incorporating them in the objective function. The objectives can be relatively prioritized.

4.3.3 Mathematical Programming Methodology

Any stochastic mathematical programming technique can be incorporated in the framework to solve the multi-metric optimization problem, provided that the probability distributions for the inter-die and intra-die variations are available. However, the evaluation and optimization of the distributions is computationally intensive. This is attributed to the fact that exhaustive Monte-Carlo simulations are required to generate the probability distributions for all the parameter variations. In several cases, appropriate empirical information is not available, thus leading to inaccurate approximation. However,

it is possible for experts to predict the pessimistic corners, and optimistic corners for the different uncertain parameters. Interval mathematics based techniques such as fuzzy mathematical programming technique can use such top level information to make better decisions in such situations. Also, Buckley [100] has shown that fuzzy programming based optimization guarantees solutions that are better or at least as good as their stochastic counterparts, since they identify the supremum of all the feasible solutions and not the averages. Thus, we choose the fuzzy mathematical programming technique as the solution methodology to illustrate our framework.

Here, we will briefly present the methodology and the formulations. Algorithm Algorithm 4.1 shows the steps involved in the process. In the fuzzy mathematical programming method [48], the parametric variations in the delay equation are modeled as fuzzy number triplets of the form (b_i, $b_i - g_i$, $b_i + g_i$) and (c_i, $c_i - h_i$, $c_i + h_i$). Here, g_i and h_i correspond to maximum variations for the coefficients b_i and c_i respectively. The coefficient b_i approximates the variation in effective channel length (L_{eff}), whereas c_i approximates the variations in oxide thickness (t_{ox}).

Algorithm 4.1 Multi-metric gate sizing algorithm

Require: Parasitic information from SPEF File, Design variables from DEF, CAP, and structural Verilog files, characterized values for nominal and worst case delay coefficients *a*, *b*, and *c*.

Ensure: Optimal Gate sizes

1: Evaluate the nominal case delay(t_{spec}) by solving a path constrained linear programming formulation, incorporating the nominal case delay coefficients in the linear delay model given in Equation (3.22)
2: Formulate and solve the nominal case multi-metric gate sizing problem through a deterministic LP formulation with noise and power objectives, and nominal case delay and noise margin constraints. Also, incorporate the spatial correlations in the modeling of the problem
3: Store the nominal case results for noise (N_{nc}) and gate sizes (S_{nc})
4: Formulate and solve the worst case multi-metric gate sizing problem through a deterministic LP formulation with noise and power objectives, and worst case delay and noise margin constraints. Also, incorporate the spatial correlations in the modeling of the problem
5: Store the worst case results for noise (N_{wc}) and gate sizes (S_{wc})
6: Using gate size values S_{nc} and S_{wc}, and the noise results N_{nc} and N_{wc}, formulate a crisp fuzzy non-linear program to maximize the variation parameter λ, under delay and noise constraints

In the worst case optimization scenario, the maximum possible variations are assumed and a pessimistic approximation is performed, and the delay equation is given by (4.6).

$$d_i = a_i - (b_i - g_i)s_i + (c_i + h_i) \sum_{j \in fo(i)} s_i \tag{4.6}$$

In the nominal case optimization, the delay equation is given as follows:

$$d_i = a_i - b_i s_i + c_i \sum_{j \in fo(i)} s_i \tag{4.7}$$

The nominal case (step 2) and the worst case (step 4) gate sizing formulations for fuzzy programming are given by (4.8) and (4.9) respectively. The spatial correlations are incorporated in the modeling by multiplying a constant multiplier to weight the variability impact of a fan-out gate on the delay of a particular gate. The farther the fan-out gate is, the lower is the weight and hence the impact of variations on that gate.

$$\begin{aligned} min \ & \sum_{i=1}^{n} (\alpha\theta + \beta\phi - \gamma\xi) s_i \\ s.t \ \ & d_p \leq t_{spec} \forall p \in P \\ & N_i \leq U_i \forall i \in n \\ & l_i \leq s_i \leq u_i \forall i \in n \\ & d_i = a_i - b_i s_i + c_i \sum_{j \in fo(i)} s_i \end{aligned} \tag{4.8}$$

$$\begin{aligned} min \ & \sum_{i=1}^{n} (\alpha\theta + \beta\phi - \gamma\xi) s_i \\ s.t \ \ & d_p \leq t_{spec} \forall p \in P \\ & N_i \leq U_i \forall i \in n \\ & l_i \leq s_i \leq u_i \forall i \in n \\ & d_i = a_i - (b_i - g_i) s_i + (c_i + h_i) \sum_{j \in fo(i)} s_i \end{aligned} \tag{4.9}$$

After the deterministic nominal (S_{nc}, N_{nc}) and worst case (S_{wc}, N_{wc}) problems are solved for the delay, noise and power values using a mathematical programming solver, the noise and gate size values are used for formulating a crisp non-linear programming model. Using a new variation parameter λ, the fuzzy optimization problem is formulated using the symmetric relaxation method [101]. The gate

sizing problem in the presence of process variations is given by (4.10).

$$
\begin{aligned}
& max \ \ \lambda \\
& \lambda(S_{wc} - S_{nc}) - GS + S_{wc} \leq 0, \\
& \lambda(N_{wc} - N_{nc}) - GS + N_{wc} \leq 0, \\
& s.t. \ \ D_p \leq t_{spec} \ \ \forall p \in P \\
& and \ D_p = \sum_{i \in p} (a_i - (b_i - g_i * \lambda) s_i \\
& \qquad + (c_i + h_i * \lambda) \sum_{j \in fo(i)} s_j)
\end{aligned}
\tag{4.10}
$$

where, the parameter λ is bounded by 0 and 1. However, for the gate sizing problem, a smaller bounds of range between 0.5 and 0.75 can be given for λ. Such a smaller bound is sufficient due to the dual requirement of high yield and low overhead for the gate sizing optimization in presence of variations, and speeds up the procedure 2-3 times, without affecting the final solution. Physically, λ can be considered as the variation resistance (robustness) property of the circuit, meaning the ability to meet timing constraint even in the presence of variations. Hence, the LP tries to maximize this variation resistance. The noise and sizing constraints ensure that the crosstalk noise and the power are between the worst case and the nominal case values. The variation resistance tries to ensure that the optimal solution values are closer to the nominal case values, and thus minimizes the power and crosstalk noise of the circuit. It has been shown in the literature [102–104] that the fuzzy non-linear programming solutions produce the most satisfying optimization results in the presence of uncertainty.

4.3.4 Paths to Nodes

An important issue in the aforementioned optimization problem is that it is intrinsically a path based formulation. This issue can be addressed by converting a path based formulation to a node based one, with each node corresponding to a gate [47]. If we consider two simple paths $a \longrightarrow c$ and $b \longrightarrow c$, where a and b are the nodes corresponding to primary inputs and c as primary output node,

then considering a dummy sink node s such that $c \longrightarrow s$, the node based formulation can be gives as:

$$\begin{aligned} & min \sum_{i=1}^{3} s_i \\ & s.t.\ d_a \leq t_c \\ & d_b \leq t_c \\ & t_c + d_c \leq t_{spec} \end{aligned} \tag{4.11}$$

where, t_c is the arrival time at c and d_is are given by the linear delay model. The sub optimality introduced due to this transformation is close to 2% for circuits with 20 levels of logic. The sub-optimality refers to the value of dynamic power obtained when compared to the path based formulation. However, this transformation significantly improves the runtime and the feasibility of optimizing large circuits.

4.4 Experimental Results

In this section, we present the experiments conducted to evaluate the performance of this multi-metric optimization framework. The framework was rigorously tested for optimization in various settings like equally weighted multi-metric optimization, single metric optimization, and adaptive multi-metric optimization where the metrics are optimized with different priorities by assigning different weight vectors to the metrics.

4.4.1 Simulation Setup

The multi-metric optimization algorithm for gate sizing under process variations was rigorously tested on the ITC'99 benchmark designs. The simulation setup had three important steps. During the first step, the RTL level VHDL net lists of the benchmark circuits were extracted for generating gate level Verilog files using the Synopsys Design Compiler tool. These gate level Verilog files and the TSMC 180nm Standard Cell libraries (LEF, TLF, DB Files etc.) are then used to place and route the designs and generate the DEF files, cell delay information etc using the Cadence Design Encounter toolbox. We synthesized the benchmark circuits using TSMC 180nm libraries.

In the second step, the parasitic resistance and capacitance information (SPEF file) was extracted from the routed designs using the Cadence Fire N' Ice RC extractor. This information is required for

extracting the coupling noise of the routed designs. The SPEF file was then used to obtain the coupling capacitance and resistance information of each net with its aggressor nets using a PERL script that was written to extract this coupling information for each net, with its top three aggressor nets. During the third step, the delay coefficients for available sizes (1x - 6x) and fan-outs of the standard cells in the TSMC 180nm standard cell library were characterized using the HSPICE simulations. Also, the variations were assumed to be 25% of the nominal values [56], and were appropriately translated to the coefficients *a*, *b* and *c*. With all the information about the delays, coupling noise etc. available for the benchmark designs, the nominal case and the worst case gate sizing optimization problems were formulated. These problems were programmed in C language, which generated the optimization models in the standard AMPL format. These extreme case problems were then solved using a Linear programming solver called KNITRO. KNITRO is a robust non-linear programming solver for both convex and non-convex optimization problems. It is specifically designed to solve problems with large dimensionality. KNITRO uses interior point and active set methods for optimization, and is capable of utilizing multiple processors. The solver is available as a part of the NEOS optimization [37] suite. The optimal solutions of the worst case and the nominal case settings are then utilized to formulate a crisp non-linear programming problem, and solved using KNITRO. A detailed flow of the simulation setup is shown in Figure 4.2.

4.4.2 Optimal Noise Margins

Ideally, the noise margin is given as the difference between the minimum sized victim and the maximum sized aggressor gates. This corresponds to the maximum coupling impact on the victim net. When the victim driver gate is sized at its minimum possible size, the signal strength on the victim net is very low. Additionally, if the aggressor net's driver gate is sized up to the largest possible size, the aggressor net's signal strength increases, thereby inducing a large cross-coupling capacitance, which can affect the victim net's signal integrity. Hence, a constraint to control the maximum tolerable noise on a net is incorporated in the model.

The gate sizes considered in this simulation setup range from $0.25e^{-6}$ to $1.5e^{-6}$. Therefore, the noise margin for a net can be given by (4.12).

$$N = (1.5e^{-6}) - (0.25e^{-6}) \rightsquigarrow (1.25e^{-6}) \tag{4.12}$$

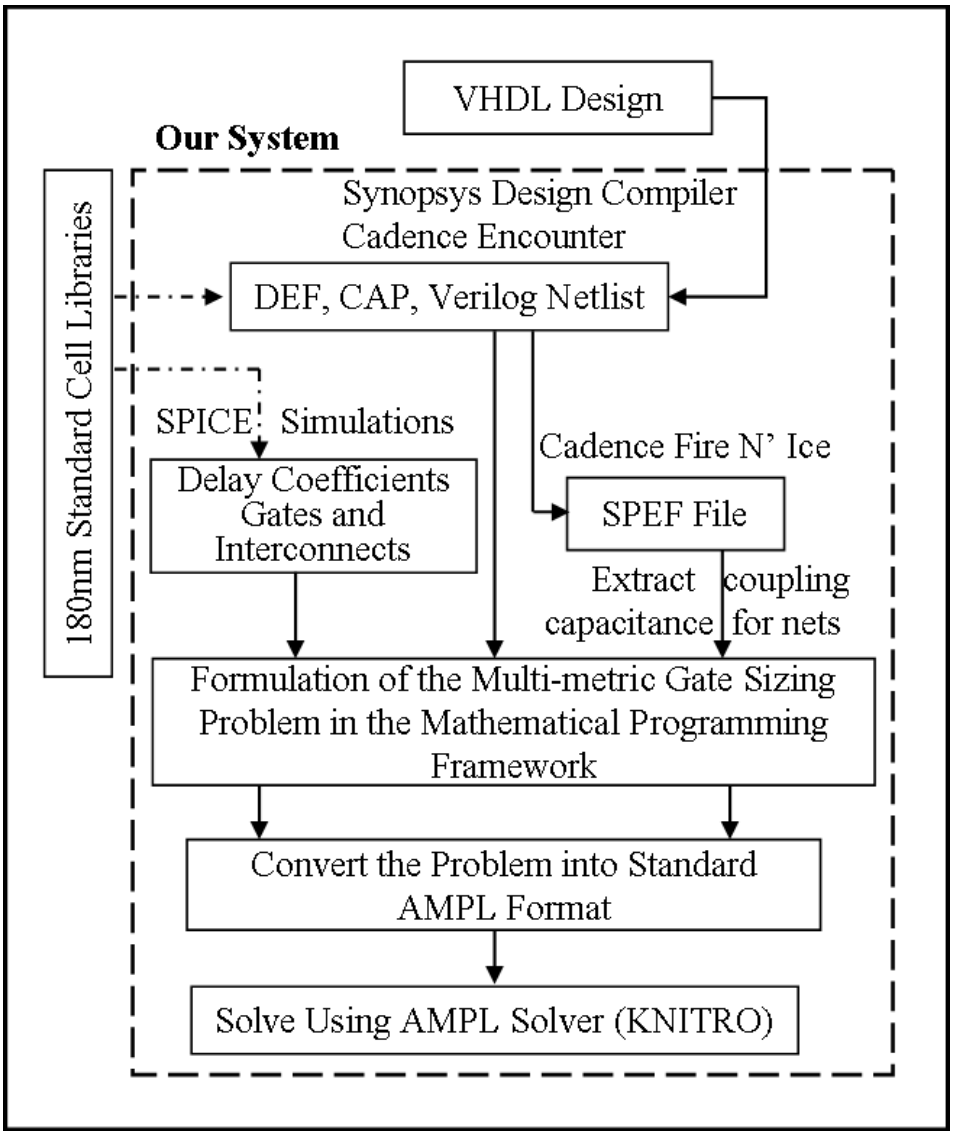


Figure 4.2 Flowchart for simulation setup. 180nm standard cell libraries have been used to extract the required files for the ITC'99 benchmark circuits.

However, these ideal noise margins are not tight, and could not capture the impact of coupling noise effectively. Thus, experiments were performed to identify the optimal noise margins, by evaluating the impact of different noise margin values on the objective functions. The experimental results are shown in Figure 4.3. Here, if the noise margin is below $0.65e^{-6}$, the dynamic power is adversely affected, even though leakage power and crosstalk noise are unaffected. Thus, after averaging the effects of noise margin on the crosstalk, and the power, a tighter noise margin constraint of $0.65e^{-6}$ was identified. This tighter noise margin can efficiently minimize the effect of coupling noise, and

generate better solution points. These noise margins are used as constraint in the (4.9) to control the crosstalk noise between the nets during the gate sizing.

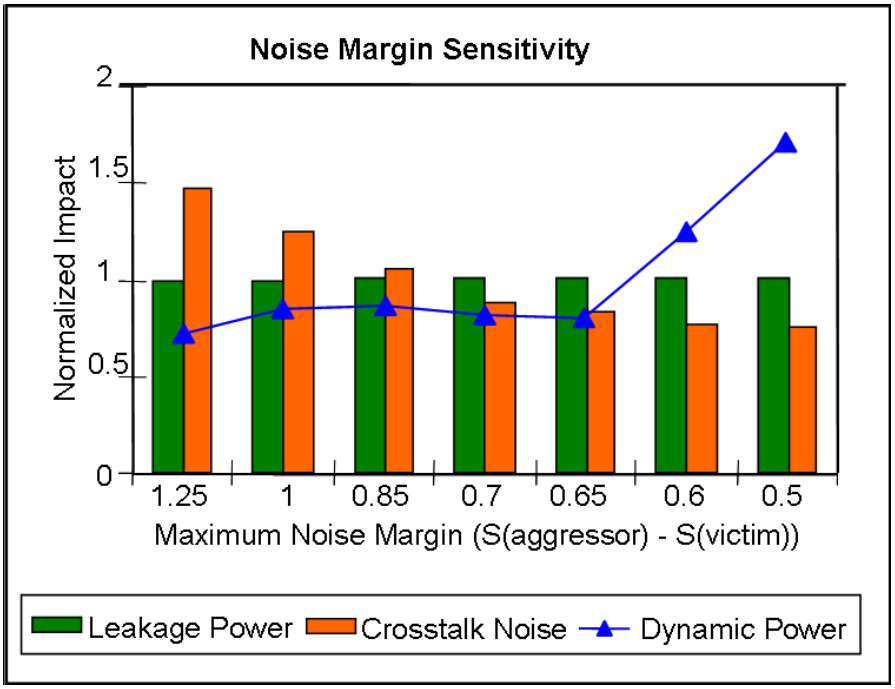


Figure 4.3 Effect of different noise tolerance values on the optimality of the objectives. Tighter tolerance values for the noise constraint derived in (4.12) have been applied and their impact on the leakage power, dynamic power and crosstalk noise has been plotted to obtain a tighter noise margin for the optimization process.

4.4.3 Determination of Timing Specification

In the first step of the optimization process, the circuit delay is optimized, since it is the primary optimization metric. A node constrained linear program is formulated using (4.1), and solved to evaluate the best timing specifications for each gate in the design. The noise margin constraints ensure that the maximum tolerable noise margins are maintained during the optimization process. However, the delay optimization results in sub-optimal values for leakage power, dynamic power and crosstalk noise. The optimal delay values identified during this step serve as the constraints for the node delays during the next steps of multi-metric optimization. The optimal delay values (t_{spec}) for the ITC'99 benchmark circuits are shown in the third column of Table 4.1.

4.4.4 Leakage Power, Dynamic Power, and Crosstalk Noise Optimization

We evaluated the optimization results when leakage power, dynamic power, and crosstalk noise are simultaneously optimized with equal priority ($\alpha = \beta = \gamma = 0.33$). The technological constraints of node delays with delay values and noise margin constraints, along with the bounding constraints corresponding to minimum and maximum available sizes of the gates are used to formulate the worst case and the nominal case optimization problems. For each of these problems, the respective characterized linear delay coefficients a, b, and c and $\bar{a_i}$, $\bar{b_i}$, and $\bar{c_i}$ are incorporated in (4.9), which was solved using the KNITRO solver. The deterministic nominal case (S_{nc}, N_{nc}) and worst case (S_{wc}, N_{wc}) power and noise results are then utilized in formulating the fuzzy mathematical program as shown in (4.10). The solution of the crisp non-linear problem obtained using the KNITRO solver gives the optimizations in the leakage power, dynamic power and crosstalk noise for the circuit.

The optimization improvements in dynamic power, leakage power, and crosstalk noise as compared to the sub-optimal values obtained during the delay optimization are shown in Table 4.1. As evident from the table, the multi-metric optimization results are significantly improved over the sub-optimal values from unconstrained delay optimization. The incorporation of spatial correlations during the modeling of the problem further eliminate the pessimism by reducing the effect of variations in circuit elements that are not in the same grid as the current element.

Table 4.1 Improvement in the optimization of metrics for multi-metric optimization with equal priority ($\alpha = \beta = \gamma = 0.33$), as compared to the values obtained during unconstrained delay optimization.

ITC'99 Benchmark	Number of gates	Unconstrained Delay t_{spec}(ns)	† Improvement in Metrics			Execution Time (secs)
			Leakage Power	Dynamic Power	Crosstalk Noise	
b11	385	0.71	12.75%	19.8%	28.1%	2.35
b12	834	0.36	14.18%	20.5%	34.76%	38.65
b13	249	0.26	57.5%	66.2%	59.98%	0.848
b14	4232	2.5	38.0%	17.92%	125.25%	177
b15	4585	3.43	25.63%	42.0%	42.35%	213
b17	21191	2.68	46.38%	57.33%	62.87%	2338
b20	8900	3.59	18.95%	21.57%	72.79%	713
b22	12128	2.63	14.57%	58.69%	59.41%	978
		Average Savings	28.49%	38.0%	60.7%	
†: Percentage improvement over the unconstrained delay optimization						

A notable aspect of this method is the runtime of the algorithm. As shown in column 7 of the Table 4.1, the runtime for the algorithm is comparatively low for even 21000 gates designs. This is attributed to the optimum modeling of the problem as a node based approach as compared to a path

based approach. Also, selection of the optimization solver plays an important role in controlling the runtime. KNITRO is a fast and accurate solver, available for both linear optimization and non-linear optimization problems.

Next, a comparative study between the pessimistic worst case analysis and the fuzzy analysis was performed to study the effectiveness of the mathematical programming technique being implemented as a solution methodology for our framework. As shown in Figure 4.4, fuzzy mathematical programming identified the solution points that significantly improved over the worst case values, and the values were closer to nominal case analysis. The improvements in the optimization are notable since the average total power savings are more than 30%, and the crosstalk noise improvement is more that 40%.

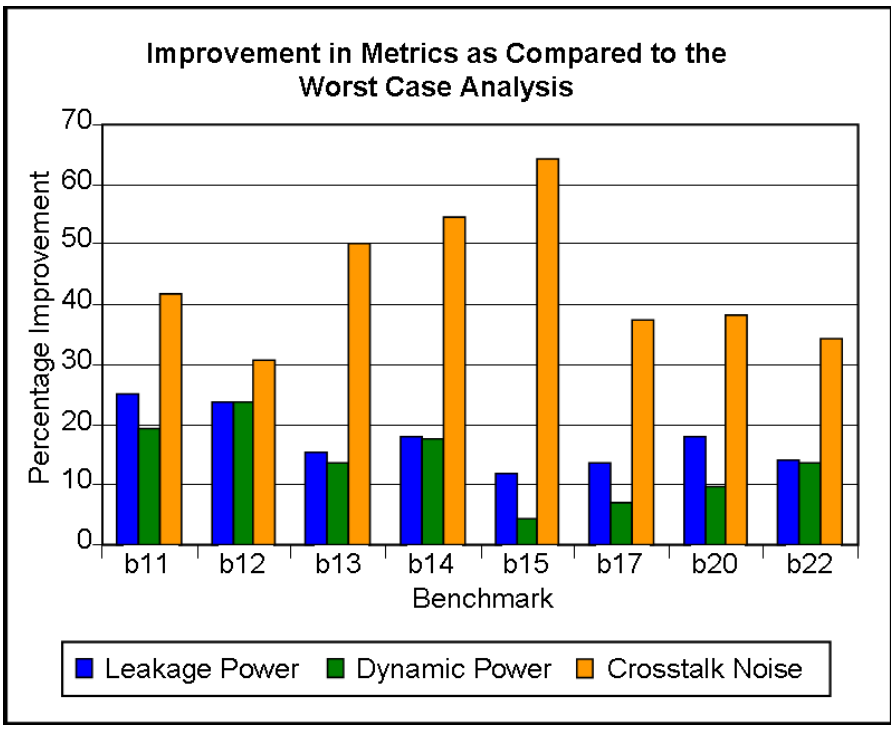


Figure 4.4 Average improvement in the metrics values for simultaneous multi-metric optimization as compared to the deterministic worst case pessimistic analysis. The metrics leakage power, dynamic power and crosstalk noise are weighted as $\alpha = \beta = \gamma = 0.33$ respectively.

4.4.5 Single Metric Optimization Results

This framework allows for selective optimization of the metrics, depending upon the design requirements. The metric to be optimized can be prioritized by assigning a high weight vector to it. For

example, if a designer intends to optimize only the leakage power, the coefficients α, β, and γ will be assigned the values as 1, 0, 0 respectively. However, when only leakage power is optimized, the crosstalk noise may be affected adversely. The impact of single metric leakage power optimization as compared to the equally weighted multi-metric optimization is shown in Figure 4.5. As shown, when only leakage power is optimized, the optimization may result in sub-optimality introduced in other metrics, like crosstalk noise in this case. This is intuitive, since the leakage power is directly proportional to the gate sizes. However, since the crosstalk noise has an inverse relationship with the gate sizes, the noise may increase as a result of optimization. Since dynamic power is affected by the size of the fan-out gates, if the gates sized during the leakage power optimization are same as the ones that affect the dynamic power, then dynamic power would also reduce. However, in the other scenario, dynamic power would be adversely affected.

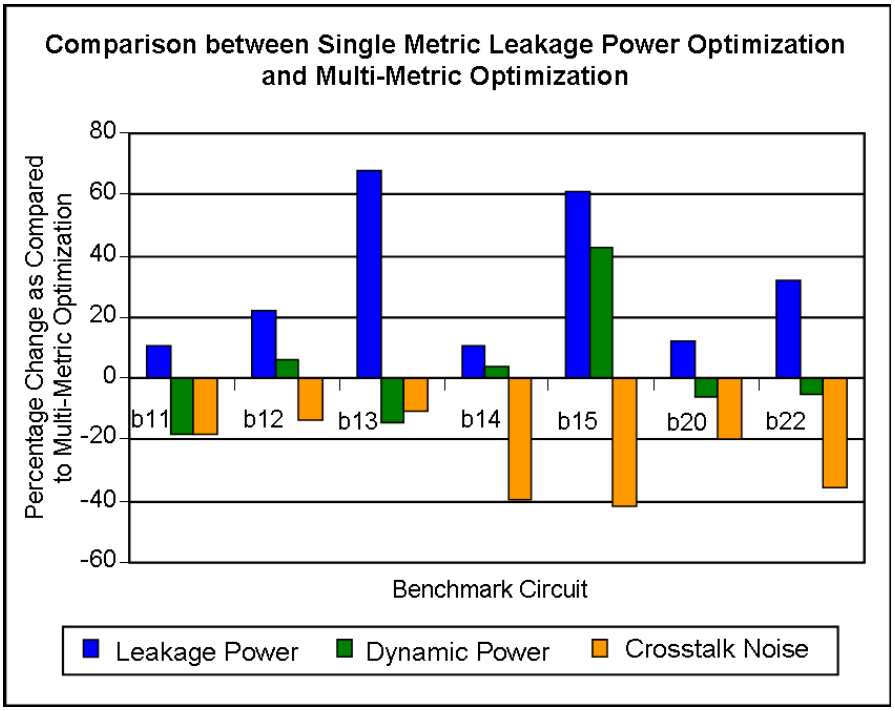


Figure 4.5 Effect of single metric leakage power optimization as compared to equally weighted multi-metric optimization. For single metric optimization the metrics are weighted as $\alpha = 1, \beta = 0, \gamma = 0$, and for multi-metric optimization the metrics are weighted as $\alpha = 0.33, \beta = 0.33, \gamma = 0.33$.

We also performed experiments for single metric optimization of the other two metrics, dynamic power (with weights $\alpha = 0, \beta = 1, \gamma = 0$), and crosstalk noise (with weights $\alpha = 0, \beta = 0, \gamma = 1$). The results for single metric dynamic power optimization as compared to equally weighted multi-metric

optimization are shown in Table 4.2. The dynamic power dissipation for single metric optimization is lower than the multi-metric optimization. However, an interesting observation in this scenario is that on the average, dynamic power dissipation is reduced at the cost of leakage power and not crosstalk noise. This trend occurs due to the fact that during the dynamic power minimization, fewer gates are resized from the sub-optimal sizes (after delay optimization) as compared to leakage power optimization. This results in a decrease in dynamic power, but the leakage power is largely unaffected.

When the single metric crosstalk noise minimization is performed, the gate sizing problem translates into a maximization problem. The results for crosstalk noise minimization as compared to the equally weighted multi-metric optimization are also shown in Table 4.2. A notable improvement (almost 2x) in crosstalk noise as compared to the multi-metric optimization scenario is identified. This is due to the fact that the maximization problem satisfies the delay constraints much easily as compared to the minimization problem. Increasing the gate sizes reduces the crosstalk noise, as well as the gate delays. However, this increases the power dissipation of the design by more than 40%.

4.4.6 Results for Priority Based Optimization

The framework can be utilized for adaptive multi-metric optimization in situations where the design requires the metrics to be optimized with different priorities. In such scenarios, the coefficients α, β, and γ are assigned weights corresponding to the relative priorities. We performed the experiments with two such scenarios. First, an optimization is performed with equal priorities assigned to leakage power and dynamic power, while neglecting the impact of crosstalk noise. The weights were assigned as $\alpha = 0.5, \beta = 0.5, \gamma = 0$. The results for optimization were compared with the equally weighted multi-metric optimization scenario to identify the percentage improvement in the two metrics leakage power and dynamic power. Since the weights were increased by approximately 17% for each metric, we evaluated if the optimality of each metric follows the same trend. The results for the percentage improvement in metrics for the benchmarks are shown in Figure 4.6. The average dynamic power improvement was 11.1% and average leakage power improvement was 12.7%. Although, the improvements were not of the same order, they were coherent with the expectations.

Finally, to compare the three scenarios discussed in this section, unconstrained delay optimization, single metric optimization, and multi-metric optimization, we compared the leakage power values

Table 4.2 Comparison of single metric dynamic power and crosstalk noise optimization with the equally weighted multi-metric optimization values.

Dynamic Power Optimization $\alpha = 0, \beta = 1, \gamma = 0$			
ITC'99 Benchmark	† Improvement in Metrics		
	Dynamic Power	Leakage Power	Crosstalk Noise
b11	13.49%	-15.11%	-12.02%
b12	8.02%	-11.73%	2.88%
b13	13.62%	-34.76%	-6.66%
b14	5.00%	-10.26	0.70%
b15	20.57%	-10.82	2.30%
b20	6.42%	-14.75	8.83%
b22	14.32%	-6.74	25.30%
Average	11.64%	-14.88%	3.05%
Crosstalk Noise Optimization $\alpha = 0, \beta = 0, \gamma = 1$			
ITC'99 Benchmark	† Improvement in Metrics		
	Dynamic Power	Leakage Power	Crosstalk Noise
b11	-19.66%	-21.34%	143.20%
b12	-20.69%	-22.38%	197.71%
b13	-35.91%	-38.40%	106.84%
b14	-15.81%	-16.11%	97.23%
b15	-19.15%	-18.88%	183.50%
b20	-19.15%	-19.81%	146.88%
b22	-10.01%	-10.19%	111.26%
Average	-20.05%	-21.02%	140.95%
†: Percentage improvement as compared to equally weighted multi-metric optimization			

obtained during the single metric leakage power optimization, and the multi-metric optimization with leakage weighted as $\alpha = 0.3$ with sub-optimal leakage power values obtained during unconstrained delay optimization. The improvements in the metric are shown in Figure 4.7. The results indicate that the single objective optimization identifies most optimal values for the metric, followed by the multi-metric optimization, which is intuitive. However, such optimizations introduce sub-optimality in other metrics like crosstalk noise and dynamic power.

4.5 Discussion

In this work, a new integrated framework for variation aware multi-metric optimization has been developed for optimization of several metrics like delay, leakage power, dynamic power, and crosstalk noise. Any mathematical programming approach can be utilized to implement this framework. In this

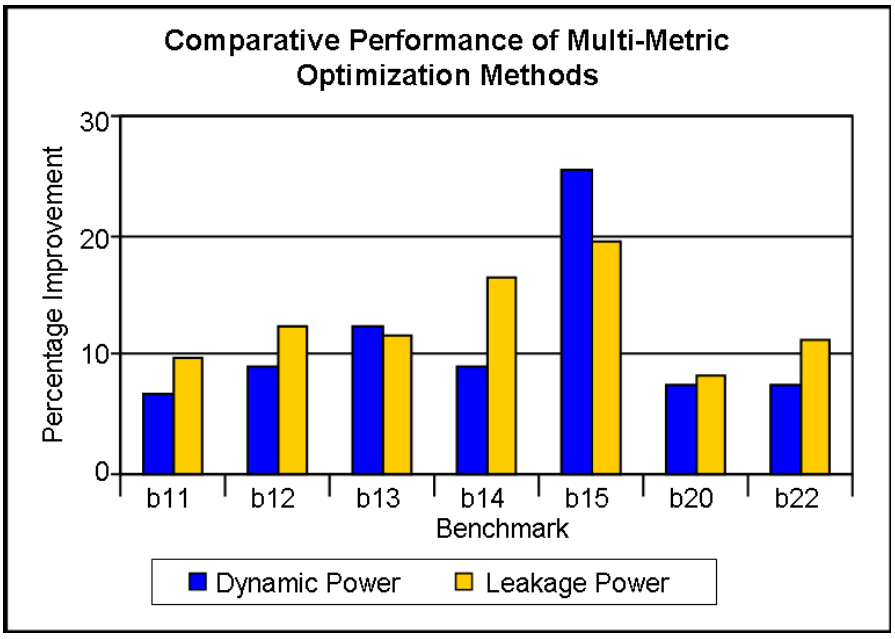


Figure 4.6 The improvement in leakage power and dynamic power when optimized with priorities $\alpha = 0.5$, $\beta = 0.5$, and $\gamma = 0$ as compared to the scenario where $\alpha = 0.33$, $\beta = 0.33$, and $\gamma = 0.33$.

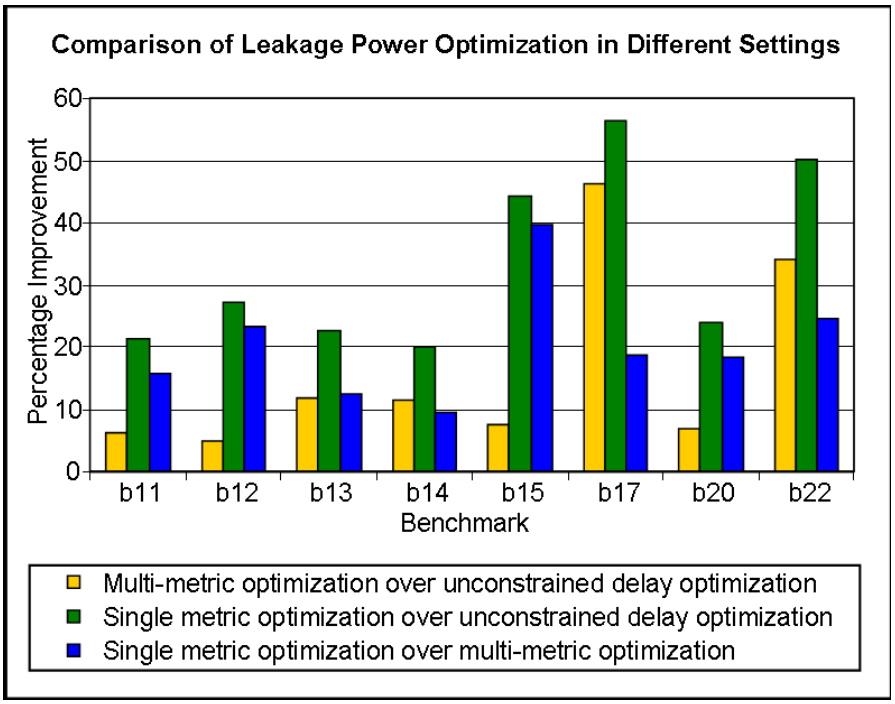


Figure 4.7 Comparative study of leakage power optimization in three different scenarios, unconstrained delay optimization, single metric leakage power optimization, and multi-metric optimization. The coefficients corresponding to the metrics in multi-metric optimization are assigned as $\alpha = 0.3$, $\beta = 0.45$ and $\gamma = 0.25$.

work, we identify the relationships between the delay, leakage power, dynamic power, and crosstalk noise metrics as a function of gate sizes and model them in a unified manner. Additional metrics like security, reliability etc. in terms of gate sizes can be incorporated in the optimization framework without any modifications. The framework is completely reconfigurable in terms of design requirements to selectively optimize one or more metrics by assigning appropriate weights to the metrics.

The experiments performed on the ITC'99 benchmark circuits indicate that the equally weighted multi-metric optimization achieves good results in terms of optimizing the values of all the metrics. Also, the weights assigned to each metric in the model are approximately linearly correlated with the average improvements in the optimization values, and hence can be used to prioritize the metrics. Although, single metric optimization achieves maximum savings for the corresponding metric, such an optimization introduces significant sub-optimality in the values of other metrics. Experiments and comparative analysis of different optimization scenarios advocate the efficacy of this framework as a generalized post-layout multi-metric optimization tool.

CHAPTER 5

A MICROECONOMIC APPROACH TO SPATIAL DATA CLUSTERING

In an optimization problem, as the size of a problem increases, the most efficient way of solving it is to form multiple partitions on the basis of certain criteria, and to solve each partition separately. However, clustering is an optimization problem, since clusters are required to be identified on the basis of specific objectives. In general spatial pattern clustering domain, several techniques have been developed in a wide variety of scientific disciplines such as biology, pattern recognition, information systems etc. While these traditional disciplines focus on developing algorithms to perform single metric clustering, various engineering and multi-disciplinary applications in emergency management, computer networks, VLSI, and robotics entail simultaneous examination of multiple metrics for spatial pattern clustering. In this work, we develop a novel multi-objective clustering approach that is based on the concepts of microeconomic theory. The algorithm models a multi-step, normal form game consisting of randomly initialized clusters as players that compete for the allocation of resources (data objects). A Nash equilibrium based methodology evaluates a solution that is socially fair for all the players. After each step in the game, the clusters are updated using any mathematical clustering algorithms. Extensive simulations were performed on several real data sets as well as artificially synthesized data sets to evaluate the efficacy of the algorithm. The experimental results indicate that our algorithm yields significantly better results as compared to the traditional algorithms. Further, the algorithm yields a high value for the *fairness index*, which indicates the quality of the solution in terms of simultaneous clustering on the basis of multiple objectives. Also, the sensitivity of the various design parameters on the performance of our algorithm is analyzed and reported.

5.1 Spatial Data Clustering

Formally, the clustering problem can be defined as an optimization problem [12, 90]: Given a set of input patterns $X = \{x_1, \cdots, x_j, \cdots, x_N\}$, a positive integer K, a distance measure δ, and a criterion

function $J(C,\delta(.,.))$ on K-partition $C=\{C_1,\cdots,C_K\}$ of X and $\delta(.,.)$, where $x_j=(x_{j1},x_{j2},\cdots,x_{jd})^T\in\Re^d$, and each x_{ji} is a feature in the feature space, partition X into disjoint sets $C_1,\cdots,C_K(K\leq N)$ such that $J(C,\delta(.,.))$ is optimized (minimized or maximized). The different clustering criteria J, and the distance functions $\delta(.,.)$ define the various clustering objectives, which may be conflicting in nature. As an example, the objectives like spatial separation and connectedness follow an inverse relationship. Similarly, the compactness of the cluster is inversely related to the equi-partitioning objective. Hence, most of the existing clustering methodologies attempt to optimize just one of the objectives that are identified to be the most appropriate in that context. This result in discrepancies between solutions provided by different algorithms and could cause a clustering method to fail in the contexts where the criterion is inappropriate.

The applications such as rescue robots deployment, ad-hoc networks, wireless and sensor networks, and multi-emergency resource management have necessitated the identification of new clustering mechanisms that could simultaneously optimize multiple objectives, which may be competitive in nature. As an example, let us consider a problem of establishment of an ad-hoc network of nodes that communicate with each other over a wireless link. Although each node has identical transmission and computing capabilities, due to power constraints, clusters are required to be created to reduce the communication overhead. Each cluster should have a cluster head that is responsible for inter- and intra-cluster communication. An optimal clustering mechanism needs to ensure that the nodes do not drop out of the network. Hence, clustering should be performed on the basis of multiple criteria; compactness for low power intra-cluster communication, and equi-partitioning for uniform power distribution. These objectives are competitive in nature and needs to be optimized simultaneously using a *multi-objective clustering technique*.

In this research, we investigate a novel methodology that identifies optimal clusters in applications with multiple conflicting objectives. This methodology consists of three components, an iterative hill climbing based partitioning algorithm, a multi-step normal form game theoretic formulation, and a Nash equilibrium based solution methodology. Specifically, in this clustering mechanism, initial clusters are identified using a mathematical approach (KMeans or KMedoids) followed by a game formulation with these clusters as players and resources. It then identifies a solution using the concepts of Nash equilibrium. Since the objectives are convex in nature, as shown in [13] a Nash equilibrium solu-

tion always exists, and tries to achieve global optima. Also, depending upon the problem formulation, the complexity of the Nash equilibrium lies between P and NP [13].

A brief review the existing clustering methodologies, the various application domains of game theory, and the applications of load-sharing are discussed in Section 2.5. The rest of the chapter is structured as follows. In Section 5.2, we discuss the motivation for identifying the approach for multi-objective clustering. The clustering methodology is presented in detail in Section 5.3. Experimental results are presented in Section 5.4, followed by a discussion on the applications and the possible future research in Section 5.5.

5.2 Why Game Theory for Clustering?

Traditionally, in data clustering, a single parameter is optimized while assuming the other parameters as constraints. However, the clustering requirements of multi-disciplinary applications have resulted in the need for new multi-metric clustering methods. In contrast to the ensemble methods that effectively integrate the results of multiple single objective clustering methods, the fundamental basis of game theory allows for the formulation of problems as multiple inter-related cost metrics competing against one another for simultaneous optimization. In game theory, each player's decision is based upon the decisions of every other player in the game, and he can optimize his gain with respect to their gains. This results in identification of global gains, and consequently an equilibrium state for the system. As an example, in the process of clustering the data objects with an objective of maximizing partition compactness, often clusters are formed such that some partitions have few objects, while others having many objects, resulting in a situation of partition-imbalance. However, a clustering performed with load-sharing or equi-partitioning as objective could result in formation of clusters with large intra-cluster distances. Thus, such situations are convex in nature, and can be successfully modeled in a game framework. Also, as shown in [13], if the payoff function in a game is convex, a Nash equilibrium solution always exists and tends to identify globally optimal solutions [105]. This is a good motivation for modeling the system in a game theoretic framework for simultaneous multi-objective clustering.

A unique property of game theory is social equity or social fairness [13], which ensures that each player in the game is satisfied and the overall goals are reached. As an example, for clustering on

the basis of three parameters, compactness, equi-partitioning, and connectedness, the other methods identify solutions targeting the global objective as a function of these design parameters. However, a game theoretic model ensures that each of these parameters is optimized with respect to the others. For an elaborate discussion of game theory the readers can refer to [27, 28].

5.3 Microeconomic Clustering Algorithm

In this section, a detailed description of the game theoretic algorithm is presented. Initially, one of the mathematical clustering methods (KMeans, in this work) is briefly explained, followed by a thorough discussion of the key components of the game theoretic model, and the model itself. An alternative ensemble based post-mathematical partitioning game theoretic method is also presented. The section concludes with the analysis of complexity of the model, and the proof of progression of algorithm.

Certain assumptions have been made during the modeling of the problem as a game theoretic framework. Most of these assumptions are not restrictive in terms of the applicability of the model, and can be discarded with no or very little changes. In this model, the objectives being considered are compactness and equi-partitioning, but the methodology is applicable to any type and number of objectives, conditional upon the convexity of the problem. The notations and terminology used in the rest of the chapter are given in Table 5.1.

5.3.1 Mathematical Partitioning

The initial set of clusters is identified using one of simplest partitioning method KMeans. This algorithm partitions a data set of size N into K clusters on the basis of minimization of the total intra-cluster variation (TICV). The steps involved in the iterative KMeans algorithm are shown in Algorithm Algorithm 5.1 .

Let $\{\mathbf{x_i}, i = 1, \cdots, N\}$ be a set of data vectors such that $\mathbf{x_i} = \{x_{i1}, \cdots, x_{id}\}$. Define a boolean w_{ik} for $i = 1, \cdots, N$ and $k = 1, \cdots, K$.

$$w_{ik} = \begin{cases} 1 & \text{if } i\text{th vector belongs to } k\text{th cluster} \\ 0 & \text{otherwise} \end{cases} \tag{5.1}$$

Table 5.1 Notations and terminology. These notations are used in the equations and algorithms described in rest of the chapter.

N	Total number of data objects in a data set
d	Dimensionality of data set
K	Total number of clusters
E_{nk}	Euclidean distance between n and k, where $n \in N$ and $k \in K$
E	Sum of the squared Euclidean distance
l_k	Number of data objects in cluster $k, \forall k \in K$
l_{ideal}	Number of data objects per cluster in equi-partitioned state; $l_{ideal} = \lvert N/K \rvert$
L	Sum of the squared load values; $L = \sum_{k=1}^{K} (l_k - l_{ideal})^2$
P	Total number of players; $P \subset K$
p_i	i^{th} player in a game;$\forall i \in P$
p_{-i}	Set of players in the game other than the player p_i
R	Total number of resource centers; $R \subset K$
r_j	j^{th} resource center in a game;$\forall j \in R$
r_{-j}	Set of all the resource centers not in the current game
U_i	Total number of strategies of a player p_i
S_i	Set of all the strategies of player p_i
s_u^i	u^{th} strategy of the player p_i; $s_u^i \in S_i$ and $u = 1, \cdots, U$
S	Strategy set of all the strategies in the game; $S = \{S_1, S_2, \cdots, S_P\}$
S_{-i}	Set of all the strategy combinations of all players other than p_i
s_v^{-i}	A strategy combination consisting of one strategy of all players other than p_i; $s_v^{-i} \in S_{-i}$

Define a matrix $W = [w_{ik}]$ such that $\sum_{k=1}^{K} w_{ik} = 1$, i.e., a data vector can belong to only one cluster (hard partitioning). Now, let $c_k = (c_{k1}, \cdots, c_{kd})$ be the centroid of kth cluster, where c_{kj} is given by equation 5.2.

$$c_{kj} = \frac{\sum_{i=1}^{N} w_{ik} x_{ij}}{\sum_{i=1}^{N} w_{ik}} \tag{5.2}$$

Then, the intra-cluster variation for kth cluster and the TICV based upon the Euclidean distance measure is given by Equations (5.3) and (5.4) respectively.

$$E^{(k)}(W) = \sum_{i=1}^{N} w_{ik} \sum_{j=1}^{d} (x_{ij} - c_{kj})^2 \tag{5.3}$$

$$E(W) = \sum_{k=1}^{K} \sum_{i=1}^{N} w_{ik} \sum_{j=1}^{d} (x_{ij} - c_{kj})^2 \tag{5.4}$$

The objective of the KMeans clustering is to identify the clusters that minimize the sum of squared Euclidean (SSE) distance measure and hence is given as,

$$E(W^*) = \min_{W}\{E(W)\} \tag{5.5}$$

Although KMeans is fast, the algorithm is sensitive to the selection of initial cluster-head positions and

Algorithm 5.1 KMeans partitioning

Require: K, data set of size N and dimensionality d
Ensure: the assignment $w_{nk}\ \forall n \in N$, where $k \in K$
1: randomly initialize K locations on d dimension space with centroids $c_k, \forall k \in K$
2: initialize iteration number $i \leftarrow 0$
3: **repeat**
4: $i \leftarrow i+1$
5: **for** $n = 1$ to N **do**
6: calculate $E_{nk}, \forall k \in K$
7: find k', such that $E_{nk'} = \min\{E_{nk}\}$
8: $w^i_{nk'} \leftarrow 1$, and $w^i_{nk} \leftarrow 0, \forall k \neq k'$
9: **end for**
10: update c_k according to equation 5.2, $\forall k \in K$
11: **until** $w^i_{nk} = w^{i-1}_{nk}, \forall n \in N$ and $k \in K$
12: **return:** $w_{nk} \leftarrow w^i_{nk}, \forall n \in N$ and $k \in K$

can easily converge to local optima if the choice of initial partitions is improper. Also, the algorithm is applicable only for single objective clustering.

5.3.2 Multi-Step Normal Form Game Model

The KMeans identifies the partitions on the basis of minimization of SSE. However, this process adversely affects the complementary equi-partitioning objective. A game theoretic methodology is described in this section to perform clustering of the data by simultaneously optimizing all the conflicting objectives. Specifically, the process involves the identification of initial clusters using the initialization step. These clusters are then categorized as players and resource centers, and a game is formalized. The players in the game compete for allocation of resources from the resource centers. The resource centers consist of a discrete set of data objects. The strategy of a player is modeled as a tuple consisting of the number of resource units requested from every resource center. The payoff corresponding to every strategy is a function of the conflicting objectives. A Nash equilibrium solution to the game is

then evaluated, and the allocations are performed accordingly. After the reallocation, the clusters are updated. This complete process is repeated until the stopping criteria are satisfied. The steps involved in the algorithm are described in Algorithm Algorithm 5.2 . The following sub-sections describe the normal form game theoretic model in details.

Algorithm 5.2 Game theoretic algorithm

Require: K, data set of size N and dimensionality d
Ensure: K partitions optimized on the basis of objectives

1: initialize K cluster centers on d dimensional space
2: perform one iteration of Algorithm Algorithm 5.1 , steps 5-10
3: **repeat**
4: $load_{before} \leftarrow getLoad(); SSE_{before} \leftarrow getSSE()$
5: **if** $\exists k \in K \mid l_k < l_{ideal}$ **then**
6: initialize a new game $G1$
7: $P \leftarrow \{m \mid l_m < l_{ideal}\}; R \leftarrow \{n \mid l_n > l_{ideal}\}$
8: **for all** $r_n \mid n = 1, \cdots, R$ **do**
9: $r_n.overhead \leftarrow l_n - l_{ideal}; r_n.consistent \leftarrow 0$
10: **end for**
11: **for all** $p_m \mid m = 1, \cdots, P$ **do**
12: perform minimum cost initial allocation of resources from R, such that $l_m \approx l_{ideal}$
13: **end for**
14: update $l_n; \forall n \in R$
15: **for all** $r_n \mid n = 1, \cdots, R$ **do**
16: **if** $l_n > r_n.overhead$ **then**
17: $r_n.conflict \leftarrow 1$
18: $G1.createStrategySet()$; %see Algorithm Algorithm 5.3 %
19: $G1.createPayoff()$; %see Algorithm Algorithm 5.4 %
20: $G1.evaluateNashEquilibrium()$; %see Algorithm Algorithm 5.5 %
21: perform temporary reallocation of units to players according to Nash equilibrium
22: **end if**
23: $r_n.conflict \leftarrow 0; r_n.consistent \leftarrow 1$
24: **end for**
25: $load_{after} \leftarrow getLoad(); SSE_{after} \leftarrow get_SSE()$
26: **if** $\%\Delta(load) > \%\Delta(SSE)$ **then**
27: commit reallocations
28: update cluster centers according to step 2
29: **else**
30: break
31: **end if**
32: **end if**
33: **until** FALSE

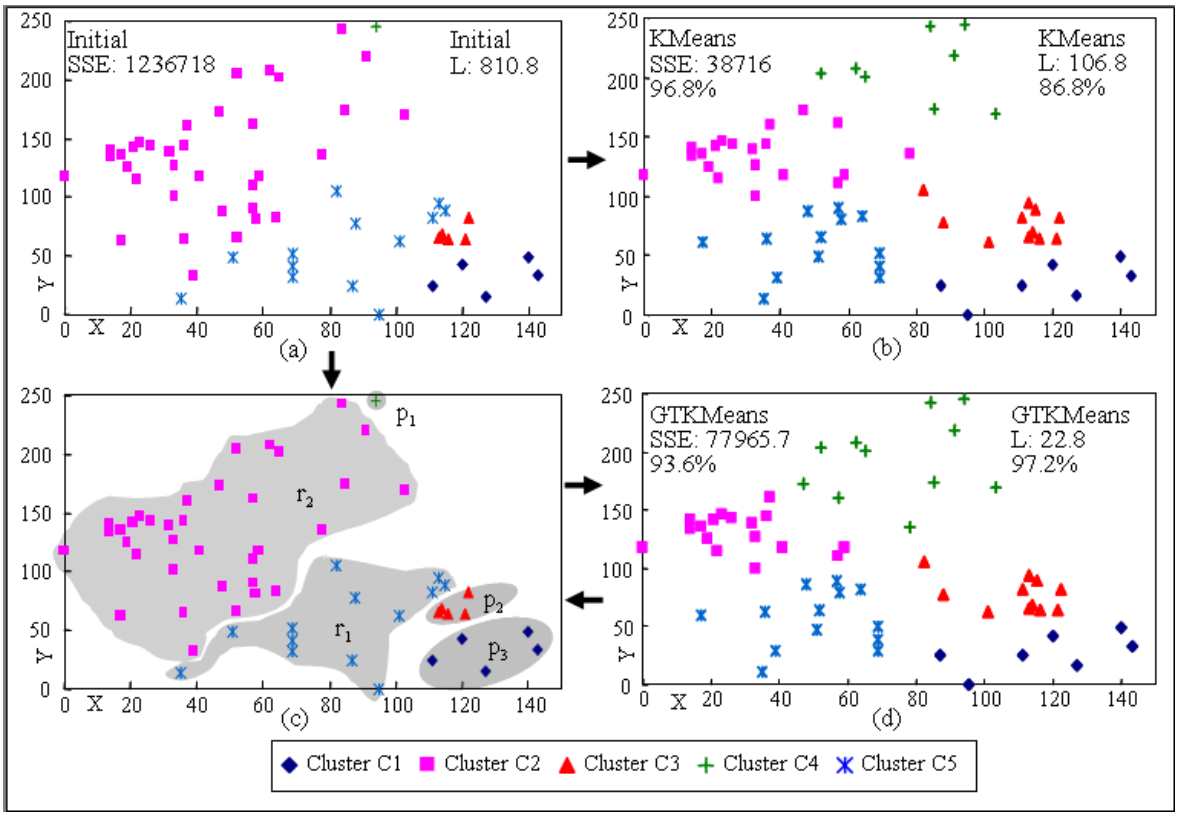


Figure 5.1 Identification of optimum clusters using game theoretic clustering (GTKMeans) and KMeans methodologies. (a)Initial clusters identified by single iteration of KMeans, (b) final clusters after KMeans, (c) formulation of a game with players p_1, p_2, p_3 and resources r_1, r_2, (d) final clusters after GTKMeans algorithm

5.3.2.1 Identification of Players

The steps involved during the algorithm can be described with the help of an example[1] given in Figure 5.1. During initialization, the cluster centers are randomly generated for the d-dimensional data set. This is followed by the identification of initial clusters by performing a single iteration of the KMeans. As shown in Figure 5.1(a), the L and the SSE values of the initial clusters is not optimal. If the iterative KMeans as shown in Algorithm Algorithm 5.1 is implemented with the objective of minimization of SSE, the final value of the SSE is 38716 (Figure 5.1(b)). However, the corresponding L value is 106.8, signifying that the clusters are not equi-partitioned. Hence, a game is formulated with the objective of simultaneous clustering of objects on the basis of compactness and equi-partitioning.

[1]The data is taken from German Town Data, which is a two dimensional data set with 59 observations, obtained from [106]. The SSE value for KMeans clustering for 5 clusters is the reported minimum value in literature [85].

The first step in the formulation of the game is defining the components of the game, i.e., the players, the resources, the strategies, the payoff functions etc. In this model, the cluster centers with $l_k < l_{ideal}, \forall k \in K$ are identified as the players in the game. Alternatively, the cluster centers with $l_k > l_{ideal}, \forall k \in K$, are considered as the resources in the game. The objective of a player is to receive the data objects from the resources in such a manner that his compactness objective and the equi-partitioning objectives are optimized simultaneously. In a situation where multiple players are requesting units from the same resource center, there is a conflict among the players, so every player competes against every other player in the game in order to maximize its own utility. One such example scenario is displayed in Figure 5.1(c), where the players p_2 and p_3 will compete to receive units from the resource center r_1.

5.3.2.2 Definition of Strategy

The feasibility of a game theoretic model largely depends upon the notion of strategy, which is a major factor in determining the computational complexity of the model. One way of defining the strategy for a player is to create a tuple consisting of the number of units that the player can request from every resource available in the system. For example, in Figure 5.1(c), the player p_3, which requires 6 resource units to realize equi-partitioned situation, could have a strategy $\{1,5\}$, i.e., receive one resource unit from r_1, and 5 units from r_2. The strategy set for the player would consist of all possible combinations of resource units from the resource locations, and the strategy space increases exponentially with every unit increase in the number of resource centers. Hence, such a notion of strategy is applicable only for the games with very few resources, and an alternate notion of strategy has to be identified for this model.

Algorithm Algorithm 5.3 outlines the steps involved in the formulation of the strategy set for a player. Essentially, it is a two step process, in which, during the first step, the players try to receive resource units from the resource locations on the basis of minimum cost allocation methodology, irrespective of the allocations made to the other players. Due to this, a situation may arise where the resource locations have allocated more resources than the overhead available with them. Therefore, for every such resource location, a game needs to be formulated and solved to ensure equi-partitioning. Hence, during step two, the cluster centers that have tried receiving resources from the resource lo-

cation in conflict are considered as the players in the game. The players' strategies consist of the number of resource units they may have to loose in order to ensure that the corresponding resource location (for which the game has been formulated) is in consistent state, i.e., the resource location is equi-partitioned. An example scenario described in Figure 5.2 would be helpful in improving the understanding. As shown in Figure 5.2, the player p_1 has requested 1 resource units from location r_1 and player p_2 has requested 4 units. Due to the requests, r_1 may loose 5 units, which would lead to a situation where $l_{r1} < l_{ideal}$. However, the players only need to loose a total of 3 units and try to receive those units from other resource locations to ensure that $l_{r1} = l_{ideal}$. So, a game is played between the players p_1 and p_2, with player p_1's strategy set as $\{0\},\{1\}$, and player p_2's strategy set as $\{0\},\{1\},\{2\}$, with the numbers indicating the resource units the players may have to loose in order to ensure that the resource center is equi-partitioned. The players would receive a payoff for every strategy, which would be a function of the additional cost incurred for receiving the resources from the centers that are farther from the player, and the change in L value for the players and the resource. Modeling of the strategy in the proposed manner reduces the strategy space considerably. Also, the number of actual players per game is significantly less than the total number of players in the system, since not all players would have requested units from the resource location that is in the conflict situation. Effectively, using this methodology, one large game is subdivided into several small games played in multiple steps.

Algorithm 5.3 Generation of strategy set

Require: resource location in conflict (r_n), $r_n.overhead$, set of all players P
Ensure: strategy set $S \mid S = \{S_1, S_2, \cdots S_P\}$
1: identify the set of players P' that received allocation of resource units from r_n
2: **for all** $p_i \mid i = 1, \cdots, P'$ **do**
3: $numstrategies = min(r_n.overhead,$ units received by p_i from $r_n)$
4: **for** $j = 0$ to $numstrategies$ **do**
5: $S_i += \{j\}$%number of units a player may have to loose from r_n %
6: **end for**
7: $S += \{S_i\}$
8: **end for**
9: return S

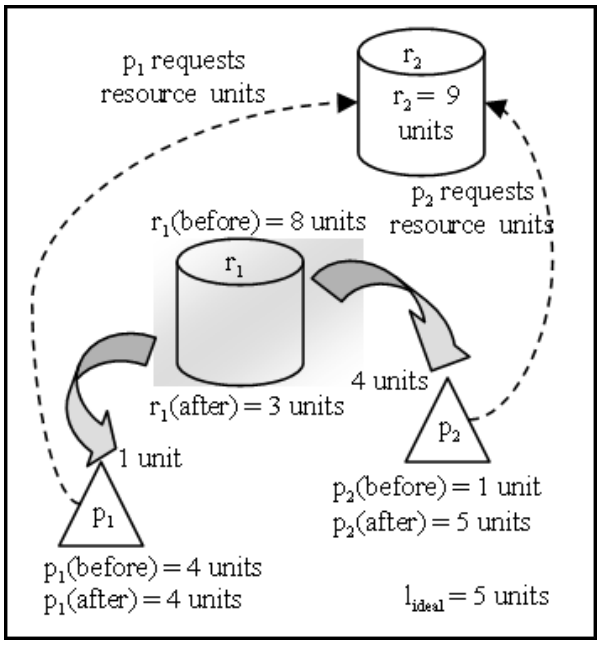


Figure 5.2 An example for definition of strategy. Here, r1 and r2 are the two clusters with more units available with them after the initial clusters are identified. p1 and p2 are the players in the game, competing with each other in order to receive maximum resource units from r1.

5.3.2.3 Payoff Function

The players in a game play their strategies in order to optimize the equi-partitioning and the compaction objectives of the resource center for which the game is played. An expected utility is associated corresponding to each strategy combination that a player in the game would receive. This utility is mathematically modeled as a payoff function, which evaluates the gain or loss a player incurs when it plays its own strategy, and the other players play their corresponding strategies. In this scenario, the payoff for a player p_i's strategy s_u^i and the players p_{-i} strategy combination s_v^{-i} for a game played for resource center r_j is affected by the following factors.

- Every resource unit that the player intends to loose from r_j is received from the other resource locations r_{-j}. This increases the SSE value for the player.
- When the other players p_{-i} in the game play s_v^{-i} before p_i's strategy s_u^i, the cost incurred for receiving the resources from r_{-j} further increases because some of the closer resource locations might have already allocated resources to the players p_{-i}.

- The load value l_{r_j} value for r_j improves as the players try to receive units from r_{-j}. However, as the total number of units lost by the players become greater than l_{ideal}, the load l_{r_j} starts worsening. Hence the absolute value of change in l_{r_j} needs to be minimized.

The payoff function captures the inter-relationship of the above mentioned criteria, and is modeled as a geometric mean of the total loss incurred by the player p_i in terms of the difference between the SSE before and after the other players p_{-i} play their strategies s_v^{-i}, and the absolute value of the load l_{r_j}, corresponding to the strategy s_u^i.

Algorithm 5.4 Payoff matrix generation

Require: strategy set S, players P', conflict resource (r_n)
Ensure: Payoff matrices po_i of players $p_i \mid i = 1, \cdots, P'$

1: **for all** $p_i \mid i = 1, \cdots, P'$ **do**
2: $rows \leftarrow \mid S_i \mid$
3: $columns \leftarrow \prod_{b=1, b \neq i}^{P'} (\mid S_b \mid)$
4: create empty payoff matrix po_i of size $rows * columns$
5: **for** $j = 0$ to $columns$ **do**
6: **for** $k = 0$ to $rows$ **do**
7: $rc_{before} \leftarrow$cost (as a distance measure) incurred to p_i for receiving k resource units from resource locations $r_m \mid m \neq n, r_m.consistent = 0$
8: $cc_{cost} \leftarrow$change in the load value of system when players p_{-i} play their strategy combination corresponding to column j, and receive resources units from locations $r_m \mid m \neq n, r_m.consistent = 0$
9: $rc_{after} \leftarrow$cost (as a distance measure) incurred to p_i for receiving k resource units from resource locations $r_m \mid m \neq n, r_m.consistent = 0$, after the other players p_{-i} have played their strategies
10: $rc_{final} \leftarrow rc_{after} - rc_{before}$
11: $cc_{final} \leftarrow \mid r_n.overhead - (cc_{cost} + k) \mid$
12: $po_i[k][j] \leftarrow \sqrt{rc_{final} * cc_{final}}$
13: **end for**
14: **end for**
15: **end for**

5.3.2.4 Nash Equilibrium Solution

The multi-objective clustering problem being modeled as a game is solved using the Nash equilibrium methodology. As compared to the other solution concepts available in the literature, only Nash equilibrium method identifies the social optima. The payoff matrices evaluated during the previous step serve as the input to the algorithm, which generates an output as a Nash equilibrium strategy set consisting of one strategy chosen for every player in the game. At the Nash equilibrium point, no

player has incentive to change its strategy unilaterally. The Nash equilibrium algorithm is explained in Algorithm Algorithm 5.5 . After the equilibrium strategies are identified, the reallocation of resource units is performed accordingly. The game is then played for other resource locations in conflict and the allocations are performed accordingly. The cluster means are then updated, and the complete process is repeated until there is no further improvement in one of the objectives without worsening of the other.

Algorithm 5.5 Nash equilibrium algorithm

Require: Payoff Matrices po_i of players $p_i \mid i = 1, \cdots, P'$
Ensure: Nash equilibrium strategy combination S^*
1: **for all** $pay_i \mid i = 1, \cdots, P'$ **do**
2: identify a strategy s_i^* such that
3: $po_i(s_1, \cdots, s_i^*, \cdots, s_{P'}^*) \geq po_i(s_1, \cdots, s_i, \cdots, s_{P'}^*)$
4: %Nash equilibrium strategy combination identified on the basis of [36]%
5: **end for**
6: $S^* = \{s_1, \cdots, s_{P'}^*\}$
7: return S^*

5.3.3 Ensemble Based Game Theoretic Clustering

As shown in the previous sub-section, the simultaneous clustering on the basis of multiple objectives is performed using multiple game iterations, where an iteration consists of multi-step games. The complexity of this method depends upon the number of data objects as well as the number of clusters, and thus the response time of algorithm is high for large data sets. Hence, an ensemble method that performs the complete clustering on the basis of fast mathematical methods followed by a game theoretic algorithm has been presented here. In this method, during the first step, a KMeans clustering of the data objects is performed on the basis of the objective of minimization of the intra-cluster distance as explained in the Algorithm Algorithm 5.1 . The clusters obtained after the KMeans algorithm are not optimal on the equi-partitioning parameter, hence a game is formulated with the players as the clusters with number of data objects less than l_{ideal}, and the resources as the clusters with the number of data objects greater than l_{ideal}. The game is then played and a Nash equilibrium solution point is identified. A reallocation of the data objects is performed if relative change in the compactness and the equi-partitioning values is below the threshold. Since, the game is played only once in this scenario, the notion of the strategy as described in the paragraph 1 of Section 5.3.2.2 can be adopted.

Although, post KMeans game theoretic model, referred as *PKGame* henceforth, does not perform simultaneous optimization of multiple objectives, the methodology is fast, and the results obtained for the experiments are promising.

5.3.4 Analysis of Game Theoretic Algorithm

In this subsection, the methodology is analyzed to evaluate its practicability. First, the computational complexity of the methodology for the extreme cases as well as the worst case scenario is identified, then some of the unique attributes of Nash equilibrium algorithm that makes it attractive as a solution method for this model are discussed. The discussion will conclude with a brief discussion about the progression of algorithm.

5.3.4.1 Computational Complexity Analysis

In a normal form P-player game with an average number of strategies S per player, the worst case time-complexity is given by $O(P*S^P)$ [36] when the game is played in single shot. However, in the model discussed in section 5.B, a multi-step game has been formulated and solved. So, the overall computational complexity of playing R such games is $O(R*P*S^P)$, where $R \subset K, P \subset K$ and $R+P \leq K$. K is the total number of clusters. Among R, P, and S, the complexity is largely governed by the value of S, which depends upon the definition of a strategy. As opposed to the natural notion of strategy as a combination of resource requests from every resource location, the strategy in this context has been defined as the number of resources a player may have to loose in order to ensure that the resource location is in consistent, equi-partitioned state. This restricts the size of strategy set of a player p_i as $\mid S_i \mid = \lfloor N/K \rfloor$. Hence, the worst case time complexity of one game is given as $K*\lfloor N/K \rfloor^K$, since $P \ll K$. Now, if the number of clusters K is 1, the computational complexity would be $R*(1*N^1) = R*N$. Similarly, if $K = N$, the complexity would be $N*1^N$, since $l_{ideal} = 1$. Therefore, for the extreme cases, the complexity of the system is $O(N^2) \ll O(R*P*S^P)$. In the worst case scenario, the number of players in the game is equal to the number of resources in the game. Hence, $K = N/2$, and the complexity of the system is given by Equation 5.6.

$$(N/2)*(N/2)*\lfloor N/(N/2) \rfloor^{N/2} = N^2 * 2^{(N-2)/2} \tag{5.6}$$

The complexity of this algorithm depends primarily on the number of games and the number of data objects in the data set. Hence, this methodology is ideally suited for multi-objective clustering in small to medium sized data sets.

The Nash equilibrium solution points possess certain attributes that make the methodology appropriate for certain applications. A Nash solution point is socially equitable, which means that every player in the system is satisfied with respect to every other player, and hence is in equilibrium. Social satisfaction is important in the scenarios where every objective in a multi-objective clustering has equal priority. Another important aspect of Nash equilibrium is that, for a mixed strategy non-cooperative game, a Nash equilibrium solution point always exist [36]. Although, a pure strategy game has been modeled in this work, the model can be easily extended as a mixed-strategy game by associating probabilities corresponding to the strategies of a player.

5.3.4.2 Nature of Algorithm Execution

The algorithm consists of multiple games, one for every resource location in conflict. The player set corresponding to a game consists of the set of clusters that have requested data objects from the resource center in conflict. Once a game is played for a particular resource location, and players receive the excess allocation from other resource locations, the location for which the game was played becomes consistent in terms of equi-partitioning. however, a situation may arise at a later time that this location again becomes inconsistent due to allocation of units to other players as a result of a game played for some other resource location. In extreme cases, this may lead to cycling, and the methodology would take infinitely long time to complete. In order to ensure, that such a situation does not occur, a *flag* is associated with every resource for which a game is played, and is set to *FALSE* initially. The *flag* is set to *TRUE* after a game is played for that resource. All the resources with *flag=TRUE* are not considered for reallocation. This ensures that the algorithm progresses in forward direction and finishes in finite number of steps. However, this may affect the quality of solution.

5.4 Experimental Results

Several single-objective clustering methodologies have been developed and employed for various applications. However, in the multi-objective clustering domain very few methods have been pro-

posed, which significantly limits the comparative study of the performance of our algorithm. The performance of our algorithm, referred as *GTKMeans* henceforth as compared to the KMeans algorithm, and a modified algorithm emulating the weighted multi-objective optimization methodology has been evaluated in this section. The first sets of experiments were performed with real data sets being used in the previous studies. To analyze the algorithm more closely in terms of efficiency and quality of the solution, artificial data sets were created to simulate the real world scenarios, and the method was exhaustively tested on those data sets. Also, the sensitivity of this method in terms of the various parameters like the number of clusters, the number of data objects per clusters and the strategy sets of the players has been investigated in this section.

5.4.1 Simulation Setup

The GTKMeans was tested on some of the data sets that have been widely used in literature for the evaluation of general purpose clustering approaches. The data sets are listed as follows:

- British Town Data (BTD): A data set consisting of four principal socio-economic data components corresponding to 50 British towns. The set was obtained from [107].
- German Town Data (GTD): A two dimensional data set containing the location coordinates of 59 German towns. The data set was obtained from [106].
- Iris Data (IRIS): A four dimensional data set consisting of the sepal length, sepal width, petal length, and the petal width measurements on 150 samples of IRIS obtained from [107].

The real data sets available in the literature often have an intrinsic structure that a specific clustering methodology attempts to comprehend and cluster accordingly. Due to this property, the clustering methods that are fitting for certain data sets may not be appropriate for others since they optimize a single objective. Hence, to better evaluate the performance of an algorithm, and analyze the sensitivity of various attributes of it, a wider range of artificial data sets need be constructed. In this work, 704 normally distributed data sets consisting of the location coordinates of data objects on a two dimensional grid of size 12*12 were created. The values of mean and variance were varied from; $\sigma = \pm 2$ and $0 \leq \mu \leq 10$. The size of data sets was varied from 50 to 150 data objects partitioned into 3 to 10 clusters. Also, intra-cluster similarity measures in terms of number of objects per cluster were

taken into consideration. As an example, a data set 6_8_90 would have 90 data objects partitioned into 6 clusters, with each cluster having the number data objects ranging from $\lfloor 0.8 * (90/6) \rfloor = 12$ to $\lfloor (0.2 * (90/6)) + (90/6) \rfloor = 15$. For each experiment, averages of 200 repetitions were performed with random cluster center (cluster head) initializations. The Nash equilibrium solution to the n-person normal form game was identified using the *Simplical Subdivision* algorithm. Among the several Nash equilibrium methodologies available in literature, the simplical subdivision method has been identified to work consistently better than other existing methodologies. The algorithm is acceptably fast for the moderate sized problems. Based upon the simplex method, the algorithm starts with a given grid size, and converges to an approximate solution point by iterative labeling of the sub-simplexes. *Gambit* [108], an open source C library of game theory analyzer software toolkit was used for identification of Nash equilibrium solution. Gambit incorporates several Nash equilibrium algorithms for solving normal form, extensive form, and Bayesian games. All experiments were performed on a Sunblade 1500 workstation that had 4 GB of RAM.

5.4.2 Experiments with Existing Data Sets

To evaluate the performance of GTKMeans algorithm, we compared it with the classical KMeans algorithm for the BTD. Since both KMeans and GTKMeans methodologies have similar starting points and both the methods identify same clusters during the initialization phase, the initial knowledge of the environment is same for both methods. Afterward, the KMeans algorithm proceeds with an objective of cluster compaction (*SSE*), whereas the GTKMeans simultaneously optimizes the compaction as well as the equi-partitioning measures (L). Figure 5.3 displays a comparative graph of GTKMeans and KMeans performance for the clustering performed on the British town data [107]. The percentage improvement in *SSE* (Y-axis on left) and L (Y-axis on right) values from the initial clusters for different cluster sizes is displayed in the graph. As evident from the graph, for $K = 4, \cdots, 10$ the percentage improvement in the L objective for GTKMeans is much higher than that of the original KMeans algorithm, whereas the percentage improvement in *SSE* is more for KMeans as compared to GTKMeans. This is due to the fact that the KMeans algorithm performs a single objective optimization only on the basis of compaction, whereas the GTKMeans algorithm identifies clusters on the basis of simultaneous consideration of both the clustering objectives. The average improvement in *SSE* and

L for GTKMeans is 87.3% and 62.7% respectively. Although the improvement in *SSE* measure is 95.8% in case of KMeans, the equi-partitioning measure improves by only 30.7%. Overall, the GTK-Means algorithm showed a mean improvement of 20% higher than that of the KMeans algorithm for simultaneously optimizing both the objective functions. To evaluate the performance of the PKGame methodology, experiments were performed on the German Town Data [106]. The performance of the algorithm in optimizing the two objectives is shown in Figure 5.4. The graph displays the relative performance of the PKGame and the KMeans algorithms. The PKGame methodology outperformed the KMeans method in terms of the average percentage improvement in the *L* for the clusters. The output characteristics were similar to the previous experiment, and an average overall improvement of 18% was noted. In an attempt to evaluate the performance of the clustering method in a multi-objective

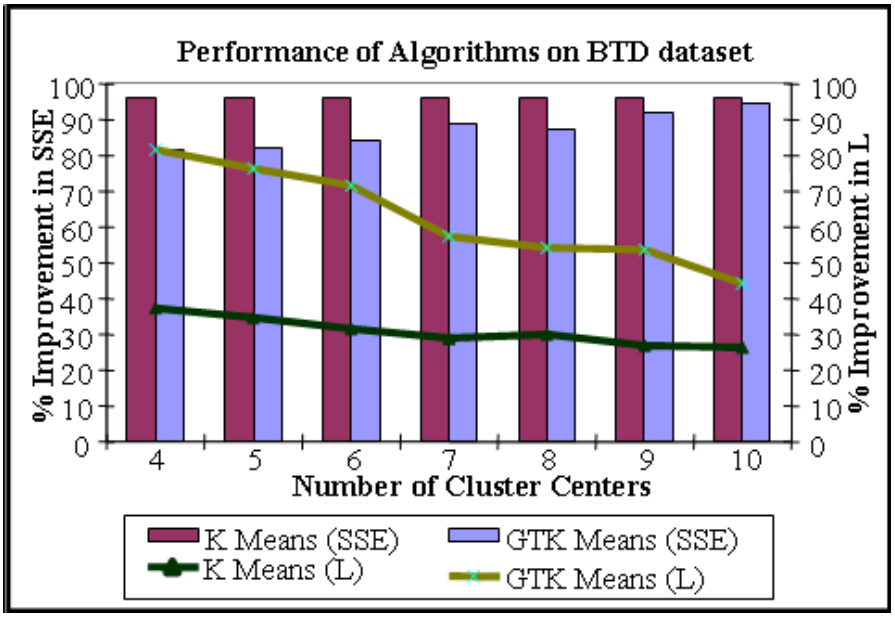


Figure 5.3 Performance of the algorithms on the British Town data set. KMeans and GTKMeans algorithms are compared on the basis of their performance in optimizing the compaction (SSE) and the equi-partitioning (L) objectives.

setting, we modified the original KMeans algorithm to incorporate the equi-partitioning objective to the original compaction objective. In this modified KMeans (MKMeans) method, the clustering was performed on the basis of a function that was a weighted average of the *SSE* and the *L* values of cluster. The weights were kept at 0.5 so that both objectives are equally represented in the solution. The results from the set of experiments performed on the IRIS data set [107] are shown in Table 5.2. The table lists the improvements in the *L* and the *SSE* values obtained after 200 iterations of GTKMeans,

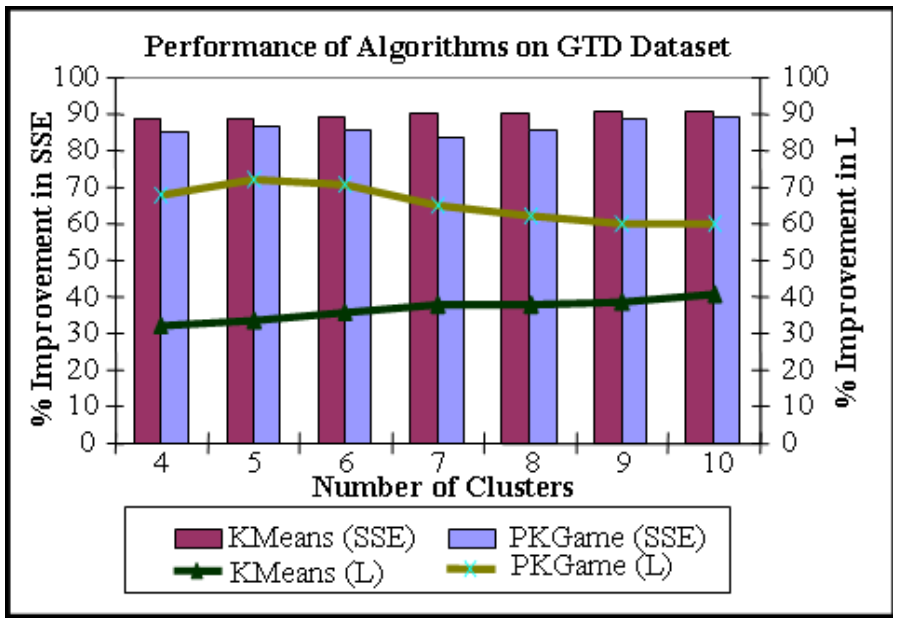


Figure 5.4 Performance of the algorithms on the German Town data set. KMeans and PKGame algorithms are compared on the basis of their performance in optimizing the compaction (SSE) and the equi-partitioning (L) objectives.

KMeans, MKMeans, and PKGame algorithms. On average the GTKMeans method outperformed other methods for majority of experiments. The ensemble based PKGame method also performed well on most of the data sets. The improvement of PKGame over the KMeans method is attributed to the fact that the former is a refinement that is performed after the latter finishes. The experiments on the existing data sets were promising, and showed the potential applicability of this method. Overall, the game theory based multi-metric clustering method outperformed the KMeans algorithm in terms of simultaneous optimization of the multiple objectives. Although, the method is slower than KMeans method in identifying clusters, it provides socially fair solutions. However, a thorough analysis of this method required further experimentation, and hence, artificial data sets were generated to evaluate the various sensitivity measures as well as the performance measures of the method.

5.4.3 Experiments with Artificial Data Sets

To evaluate the performance of the two microeconomics based methods the multi-objective clustering was performed on the artificial data sets described in the beginning of this section. An average of the outputs for improvements in *SSE* and *L* values were plotted on graphs as shown in Figure 5.5 and Figure 5.6 respectively. From Figure 5.5, it can be identified that the KMeans algorithm performs

better than the game theoretic methods for the compaction objective. Also, the performance of the MKMeans method follows the KMeans closely. This behavior is intuitive as the means based partitioning methodologies optimize only the *SSE* attribute. However, from the Figure 5.6, it is evident that the performance of KMeans for equi-partitioning objective is significantly inferior as compared to the GTKMeans and PKGame methods. This follows from the fact that the two objectives are often inversely correlated, and the unilateral improvement in one objective function adversely affects the other objective. However, since the GTKMeans method simultaneously optimizes both the objectives, the clustering performance was improved by more than 50 percent for both the objectives, as shown in the graphs. Another observation was that the performance of the ensemble based PKGame method did not improve much for the smaller clusters, i.e. $K = 3,4$, but for the larger number of clusters, the ensemble method also performed well. Since the KMeans works very well for smaller number of clusters (3-4), the compactness values are high (also evident from graph), and hence, when game theoretic method is applied after KMeans, the L improves at the cost of *SSE*, which is not desired.

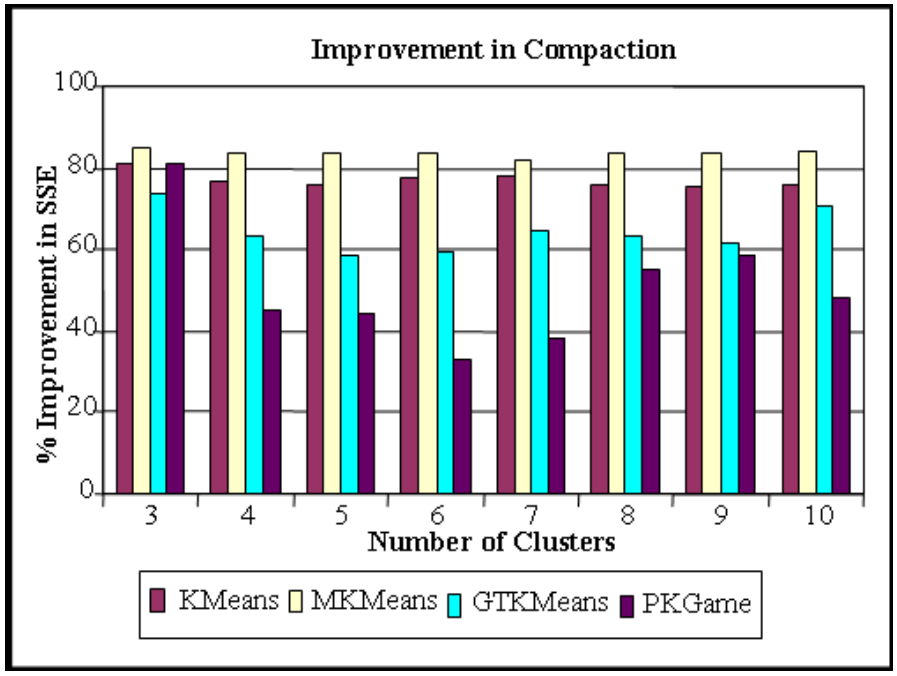


Figure 5.5 Average improvement in the compaction objective for the experiments on artificial data sets. The optimization in the *SSE* metric compared to the worst case values is evaluated for KMeans, GTKMeans, MKMeans and PKGame.

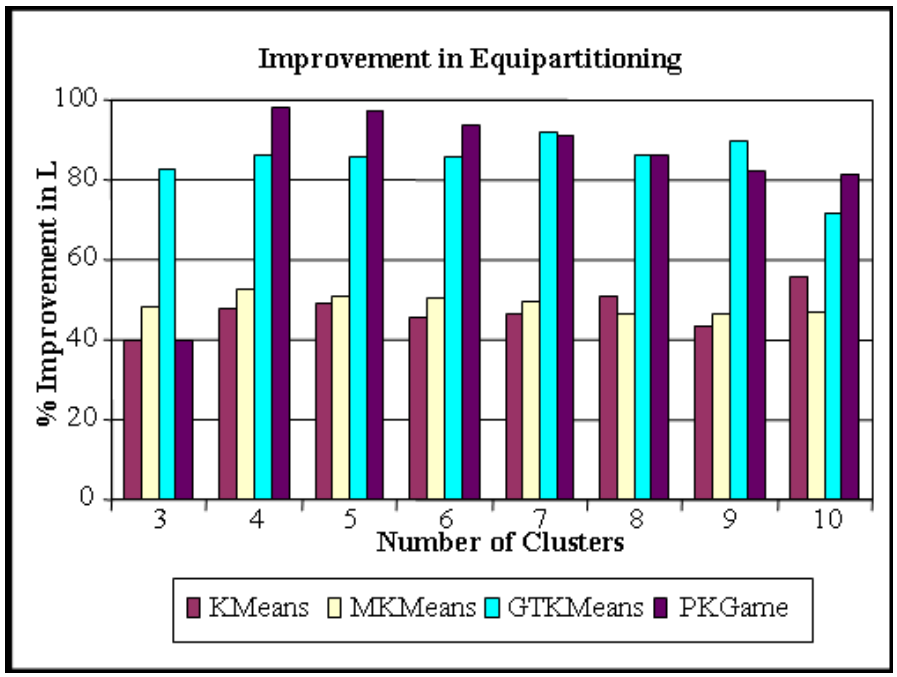


Figure 5.6 Average improvement in the equi-partitioning objective for the experiments on artificial data set. The optimization in the *L* metric compared to the worst case values is evaluated for KMeans, GTKMeans, MKMeans and PKGame.

5.4.4 Fairness of Clustering

The strength of the game theoretic clustering methodology lies in the fairness of optimizing each objective with equal priority. To appropriately evaluate the performance of the algorithms, a quantitative measure of the fairness of the algorithms for optimizing *SSE* and *LOAD* can be identified using the *Jain's Fairness Index* [109], or a geometric mean of the relative improvement in the clustering criteria. According to the Jain's index, the fairness of the methodology is identified using Equation 5.7.

$$fairness = \frac{(\sum_{i=1}^{n} x_i)^2}{(n * \sum_{i=1}^{n} x_i^2)} \tag{5.7}$$

Here, x_i corresponds the improvement in the i^{th} objective. The fairness value ranges from 0 (worst case) to 1 (best case). Similarly, The geometric mean of the improvements in the clustering criteria identifies the average performance of the methodology, equally weighing all the criteria. Table 5.3 shows the fairness metric values for different number of clusters. As shown, the GTKMeans method has a high Jain's fairness index averaging 0.98 as compared to the KMeans value of 0.93. This signifies

that the GTKMeans method optimizes both the objectives with equal priority. Similarly, the geometric mean of the GKT-Means is higher than the KMeans by more than 15 percent. The fairness performance of the MKMeans method and the PKGame method is also inferior to the GTKMeans fairness.

5.4.5 Sensitivity Analysis

The experimental results on the artificial data sets, shown in the previous subsections give hints about the sensitivity of this methodology for different design attribute values. In this subsection, we will closely analyze the sensitivity of the GTKMeans method. The number of players, number of strategies per game, response time of the algorithm, and structure of the data set significantly affect the practicability of this method. In the following subsections, we experimentally analyze these parameters.

5.4.5.1 Data Set Similarity Measure

In many cases, the structure of the data set has a significant impact on the performance of an algorithm. We generated a wide range of artificial data sets in terms for number of data objects per cluster defined as the similarity measure, and radius of a cluster as $\sigma = \pm 2$ on a 10*10 grid. The effect of structure on the execution time of the algorithm for different similarity measures and cluster sizes is shown in Figure 5.7. As shown, the similarity measure does not significantly impact the performance of the algorithm, i.e., on average, the execution time of the GTKMeans algorithm is independent of the structure of the data set, and hence it is suitable as a general clustering methodology. The average performance in terms of fairness of allocation is shown in Table 5.3. The geometric mean fairness is in range 60-80 percent, which is a good measure of fairness. Hence, the structure of a data set does not adversely affect the performance of this methodology.

5.4.5.2 Number of Players and Strategies

An important consideration during the modeling of a problem in a game theoretic framework is the impact of the size of game. The size determines the complexity, and consequently the performance of the system. Thus, we evaluated the average size of the game in terms of the number of players and the strategies for different clusters. The graph shown in Figure 5.9 displays the range of players and

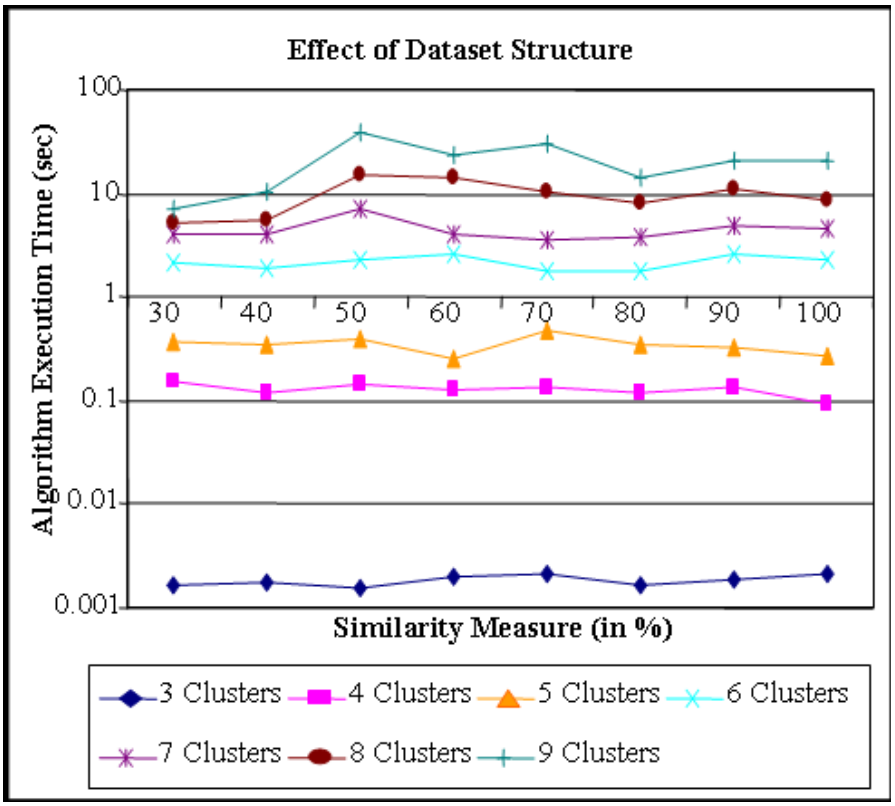


Figure 5.7 Effect of data set similarity measure on the execution time of the GTKMeans algorithm. The initial data set similarity measure is given as the degree of similarity in the sizes of the initial clusters. A higher degree of similarity results in initial clusters with almost equal number of data units per cluster.

consequently the strategies for different clusters. An important observation is that although the average number of players increases as the cluster size increases, the total number of players is significantly less than half the cluster size, which is the worst case scenario. For example, on average there are at most 3.5 players for the data sets with 9 clusters. It is also important to note that the average strategy size does not increases exponentially as a function of the number of players, which is the intuitive notion in a game theoretic setup. This behavior is attributed to the alternative definition of a player and strategy for our model as discussed in section 5.3.2.2. The modeling controlled the complexity of the system significantly. However, the surge in the number of strategies for data sets with large number of clusters indicate that the GTKMeans is better suited for multi-objective clustering of medium sized data sets with a less number of clusters per data set.

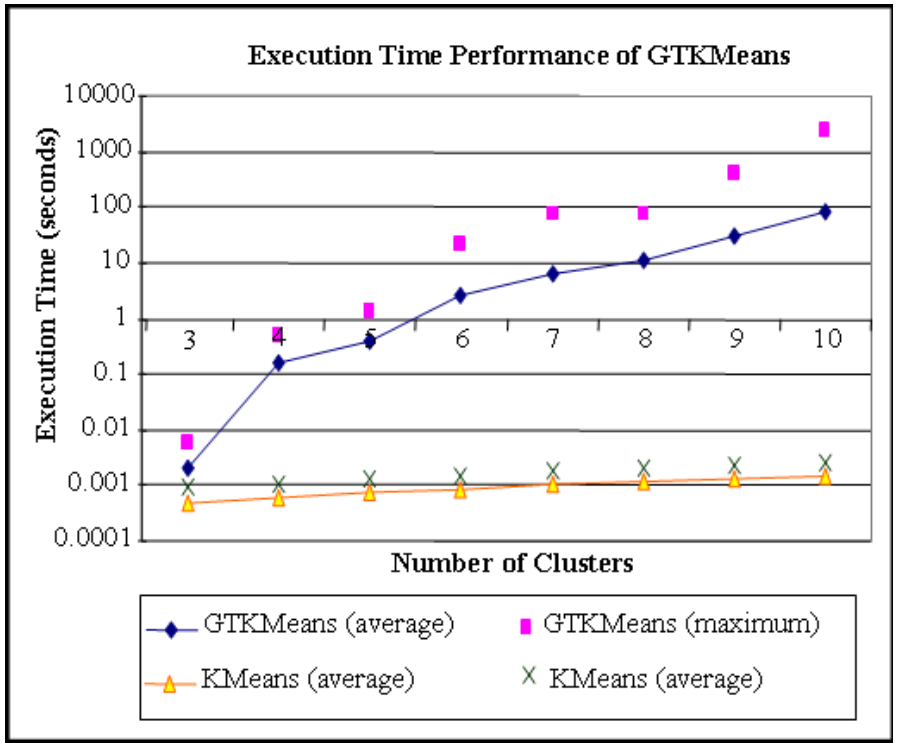


Figure 5.8 Relationship between the execution time and the number of clusters. The algorithm execution time of KMeans and GTKMeans are compared in this set of experiments. Additionally the worst case and the average case execution times are plotted and compared.

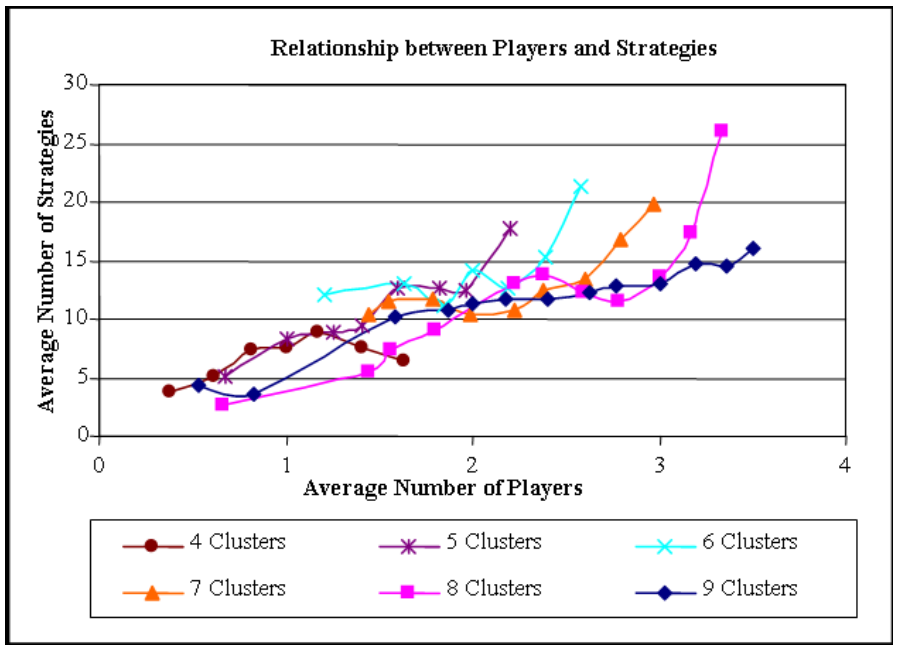


Figure 5.9 Average number of players and strategies for different cluster sizes. For different cluster sizes, the average number of players and strategies per game indicate the size of the game and consequently the execution time and the feasibility of the clustering algorithm.

5.4.5.3 Execution Time

The multi-objective clustering methodology presented in this work is slower than the KMeans method by multiple orders of magnitude. Similar is the case with other heuristics based methodologies. In order to quantify the effect of number clusters on the execution time of the algorithm, and analyze the performance extremes, we plotted average execution time and the maximum execution time for different number of clusters. As shown in Figure 5.8, for smaller number of clusters, i.e., $K = 3, \cdots, 8$, the GTKMeans performs well and identifies the optimum clusters within 10 seconds. Also, the worst case performance follows similar trend and is within 100 seconds. However, for larger number of clusters, the performance decays exponentially. This is due to the fact that as the number of clusters increase, the potential number of clusters and correspondingly the strategies increase significantly, and the game becomes large. The time complexity of the Nash equilibrium algorithm is exponential, which results in slower execution time for such cases.

5.5 Discussion

A novel microeconomics based algorithm for multi-objective clustering problem has been developed in this research. In this algorithm, a non-cooperative multi-player normal form multi-step game is formulated with the subsets of initially identified clusters as players. Any mathematical partitioning method can be employed to identify the initial clusters and to update the clusters after an iteration of the game. A Nash equilibrium based method is used to solve the game theoretic formulation. This algorithm is independent of the type and the number of objectives that can be simultaneously optimized. Also, an ensemble based game theoretic optimization algorithm has been developed in this work. In the ensemble based method, the KMeans partitioning is performed first, followed by a game theoretic formulation based upon the sizes of the clusters. The experimental study on the existing and artificial data sets provides important insights for the game theoretic clustering algorithm. As compared to the KMeans, this algorithm performs significantly better in terms of the fairness toward improving the clustering criteria. Also, the complexity of the algorithm in terms of players and strategies is much lower as compared to the classical normal form game theoretic modeling. This is attributed a novel definition of strategy. This algorithm is not sensitive the structure of the data set. However, the algorithm does not scale very well with the size of the data sets in cases where the number of

clusters increase. Overall, this model is well suited for a multi-objective clustering problem where the objective functions are complementary and need to be optimized simultaneously.

The domain of multi-objective clustering is receiving significant attention as the newer multidisciplinary research areas are emerging. This first attempt in propounding a game theoretic solution is attractive. The applications of this algorithm may require several objectives to be considered simultaneously, depending upon the application area. Also, an alternate modeling of the payoff function may improve the cost function in terms of capturing the essence of competitive objectives, and thus need further investigation and refinement. A logical next step in research is to model this game theoretic clustering approach for dynamically changing scenarios. Similarly, techniques for pruning the strategy sets would also result in multi-fold improvement in the performance and complexity of the model.

Table 5.2 Performance of the algorithms on Iris data set. The clustering algorithms KMeans, MKMeans, GTKMeans, and PKGame are compared for their performance on two optimization metrics, compaction (*SSE*) and equi-partitioning (*L*).

Total Clusters	Average Improvement in SSE† Value (in %)				Average Improvement in L‡ Value (in %)			
	KMeans	MKMeans§	GTKMeans¶	PKGame††	KMeans	MKMeans	GTKMeans	PKGame
2	94.5	93.9	84.3	94.5	45.8	40.3	57.1	45.8
3	93.7	97.4	76.8	93.8	35.9	75.9	57.6	35.9
4	93.6	98.2	72.4	86.1	35.6	64.5	94.2	99.9
5	93.6	98.3	75.4	90.4	33.6	55.8	82.0	97.9
6	93.2	98.7	67.2	91.4	30.1	55.2	61.7	91.4
7	93.4	98.8	63.9	93.6	29.5	44.7	75.2	90.8
8	93.9	98.9	51.9	94.9	30.8	51.7	59.4	82.9
9	93.8	99.0	59.8	95.2	29.6	45.7	60.2	83.4
10	94.2	99.0	72.2	95.0	29.0	43.1	69.9	86.1
11	94.7	99.1	66.3	95.7	31.8	43.0	66.2	85.1

†: Sum of Squared Euclidean Distance (SSE) corresponds to the compaction objective
‡: Load(L) corresponds to the equi-partitioning objective
§: Modified KMeans (MKMeans) algorithm
¶: Game Theoretic KMeans (GTKMeans) algorithm developed in this research
††: Post KMeans Game Theoretic (PKGame) ensemble based algorithm developed in this research

Table 5.3 Fairness of the clustering algorithms. The KMeans, MKMeans, GTKMeans and PKGame algorithms are compared on the basis of the quantitative measure of the fairness of the clustering. The two fairness indexes used for the comparison are Geometric mean fairness index and Jain's fairness index.

Clustering Algorithm	Geometric Mean Fairness Index								Jain's Fairness Index							
	3	4	5	6	7	8	9	10	3	4	5	6	7	8	9	10
KMeans	57.1	60.5	61.2	59.4	60.1	60.9	56.9	62.0	0.89	0.94	0.95	0.93	0.93	0.95	0.93	0.96
MKMeans	64.0	65.9	65.2	64.9	63.7	62.2	62.0	62.7	0.92	0.94	0.94	0.94	0.94	0.92	0.92	0.92
GTKMeans	78.0	73.8	72.7	71.2	77.1	73.8	74.3	76.2	0.90	0.98	0.98	0.97	0.97	0.98	0.97	0.99
PKGame	57.1	66.5	56.7	46.3	36.5	41.3	41.8	44.9	0.90	0.88	0.80	0.72	0.66	0.72	0.74	0.78

CHAPTER 6

GAME THEORETIC APPROACH TO ROBOT TEAM FORMATION

The aggregation of robots into teams is necessitated due to the limited power and communication capabilities in emergency environments. The formation of teams of robots significantly enhances the performance and efficiency of search and rescue missions in such environments. As opposed to the classical partitioning application domains, the robot aggregation requires multiple conflicting objectives to be optimized. We present a new method for simultaneous multi-objective partitioning of robots into teams, which is based on the concepts of microeconomics. The method utilizes the strengths of KMeans algorithm, game theoretic modeling, and Nash equilibrium methodology for fast and socially fair partitioning. In this work, partitions are created to identify decentralized teams of robots in such a manner that each robot in a team closest to its communication gateway, as well as each team is equally represented in terms of its strength (battery power). Rigorous simulations were performed to evaluate the performance of the method, and the results indicate that our method performs significantly better than the KMeans methodology, and identifies good solution points.

6.1 Problem Description

In the recent years, search and rescue robotics has emerged as an important emergency response function. Mobile robots have been shown to be a valuable resource during the exploration missions in the event of such emergencies [110]. These robots are involved in collecting and integrating the information, and transmitting it to the base station for further deliberation. In a centralized system, this requires each robot to maintain a wireless connection with the base station and constantly send the information packets. However, this communication is significantly limited by the strict constraints of battery power, low radio range, and constantly changing environment for every robot.

For a detailed explanation of the steps involved in the multi-emergency robot deployment and the issues faced in the process, please refer to Figures 6.1 – 6.5.

- Figure 6.1 shows a scenario where multiple emergency situations have emerged in a locality in a time-overlapped manner. Often in such situations, the deployment of emergency response personnel is not feasible, and hence robotic units play an important role in the search and rescue missions.

- The robots deployed in the field requires two types of communication. Each unit needs to communicate with the base station to receive the command and control. The feedback from the emergency location is continuously transmitted to the base station. Also, the robots communicate with each other in order to coordinate the coverage area among them. This ensures that the complete terrain is covered. The deployment of robots and the interconnection network being established in such scenario is shown in Figure 6.2.

- However, a point-to-point grid based networking scheme where each node communicates with every other node and the base station is not feasible in these scenarios. This is shown in Figure 6.3. Due to the limited battery power and high communication overhead, a few robotic units may drop out of the system as the time progresses, resulting in a situation where the communication with some of the emergency location would be lost.

- Thus, a partitioning mechanism may be used to form teams of robot units, such that each partition has a set of robots that are close to each other and hence dissipating less power in intra-cluster communication. Also, a partition head is decided among the nodes of the partition (possibly the one with maximum available battery power) which is responsible for the inter-cluster as well as the cluster to base station communication. In this manner, the communication overhead is reduced and the robots may sustain in the field for longer duration. If a classical clustering scheme like KMeans is used for partitioning, the teams are formed as shown in the Figure 6.4.

- However, the partitioning requirements in a multi-emergency rescue and response are different from other environments, in the sense that the partitioning is required to be performed on the basis of multiple criteria. In this particular case, the teams of robots being formed should possess two important properties; the intra-cluster communication should be minimized to reduce the power dissipation in communication, and each team should be equally represented in terms of

its capabilities. The set of capabilities could be anything ranging from the equal distribution of the robotic units in terms of their rescue capabilities, the distribution of robotic units that are specialized to perform certain jobs, or a simple equal distribution of the total battery power in each cluster to ensure that each emergency location is examined with equal capabilities. The classical partitioning algorithms are largely single metric optimization methods, and thus can not be used for partitioning. They often result in formation of partitions that are either too large or too small. One such partitioning result is shown in Figure 6.5, where a couple of partitions are too large and a couple of them are too small. If the partition is too small, the robots in that partition will have to perform all the work, as well as communication, and will soon drop out of the network due to rapid power dissipation.

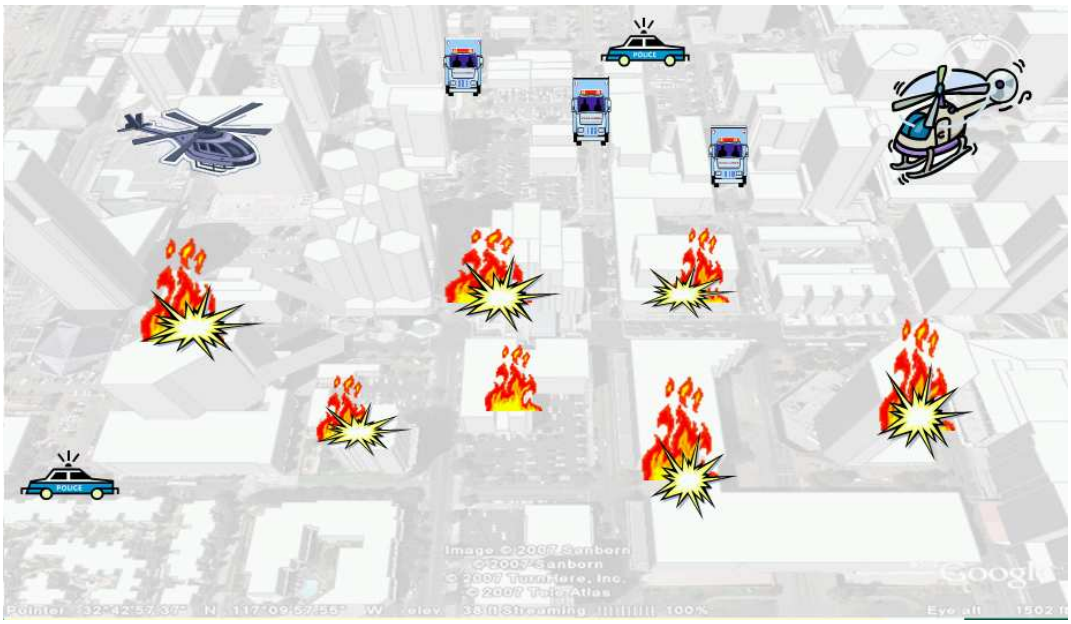


Figure 6.1 Example of a multi-emergency situation in a suburban area. Several emergency management resources are allocated to the emergency locations for search, rescue, response and recovery process.

Although, the issue of work distribution to the robots within a team [111, 112] has received significant attention, the development of specialized algorithms for optimal aggregation of robots into teams has not been explored. Unlike classical application domains like data mining, bio-informatics, computer vision and pattern recognition, computer and communication networks, and information systems [11,12], where object and data clustering are performed on the basis of single objective, multi-disciplinary applications like robot team formation require multiple criteria (that may be conflicting

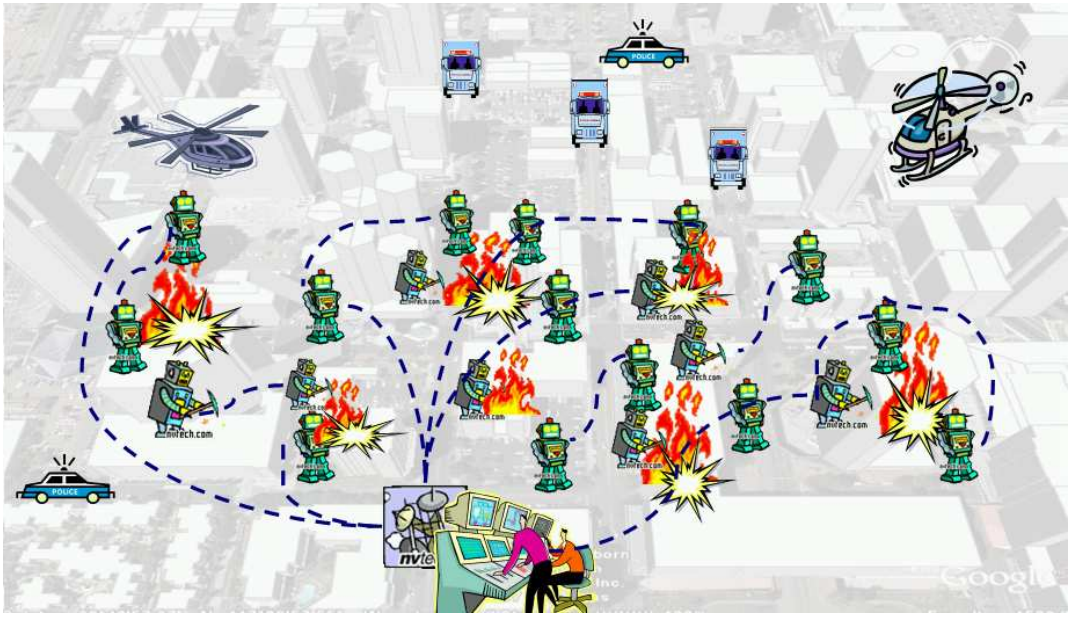


Figure 6.2 An example of search and rescue robot deployment in multi-emergency scenario. The robotic units are deployed in situations where search and rescue situations are complex and inaccessible to humans. A primitive inter-connection network is established to monitor progress in real time and share the information among the robots, and between robots and the base station.

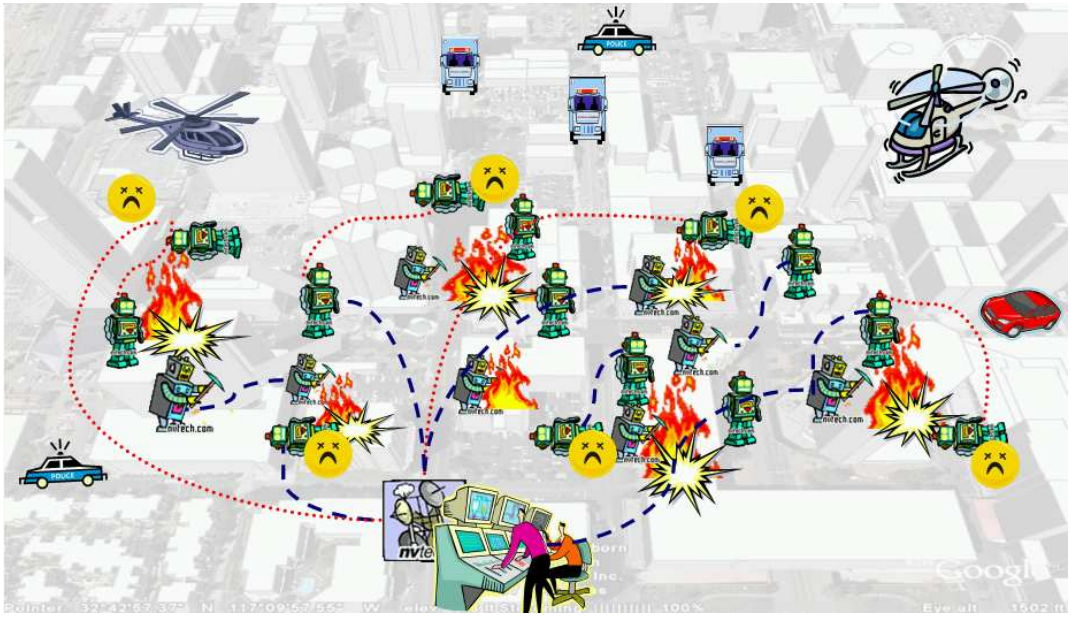


Figure 6.3 Effect of high communication overhead on the search and rescue process. As a result of high communication bandwidth and the limited battery power of each robot, the robotic units may die, thereby disrupting the response from some emergency locations.

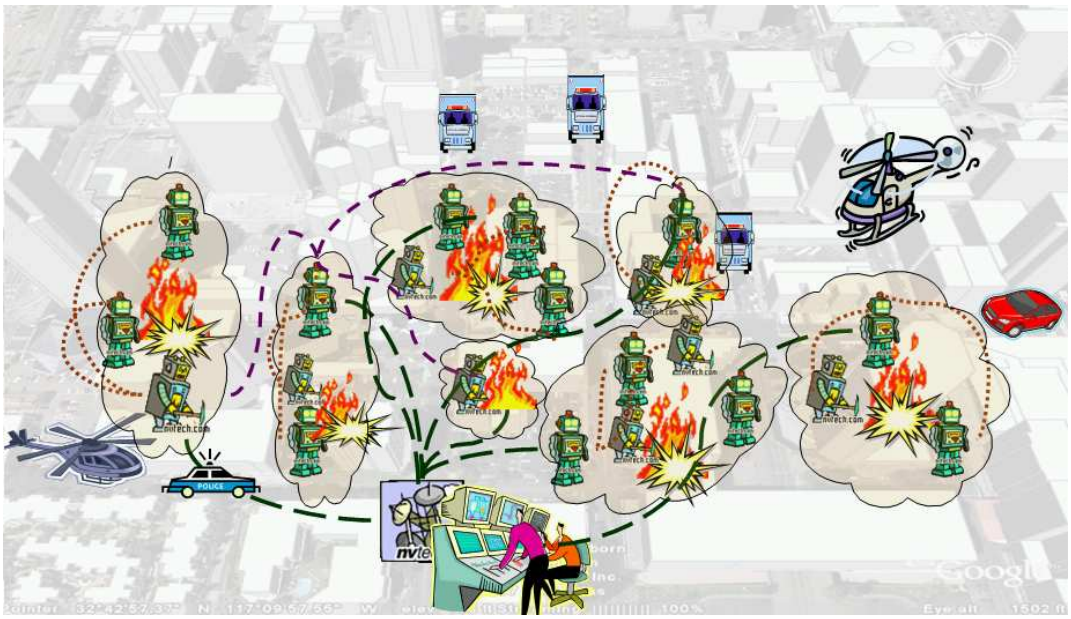


Figure 6.4 Partitioning of robots such that the intra-cluster communication is minimized, and each partition has a head node responsible for inter-cluster communication.

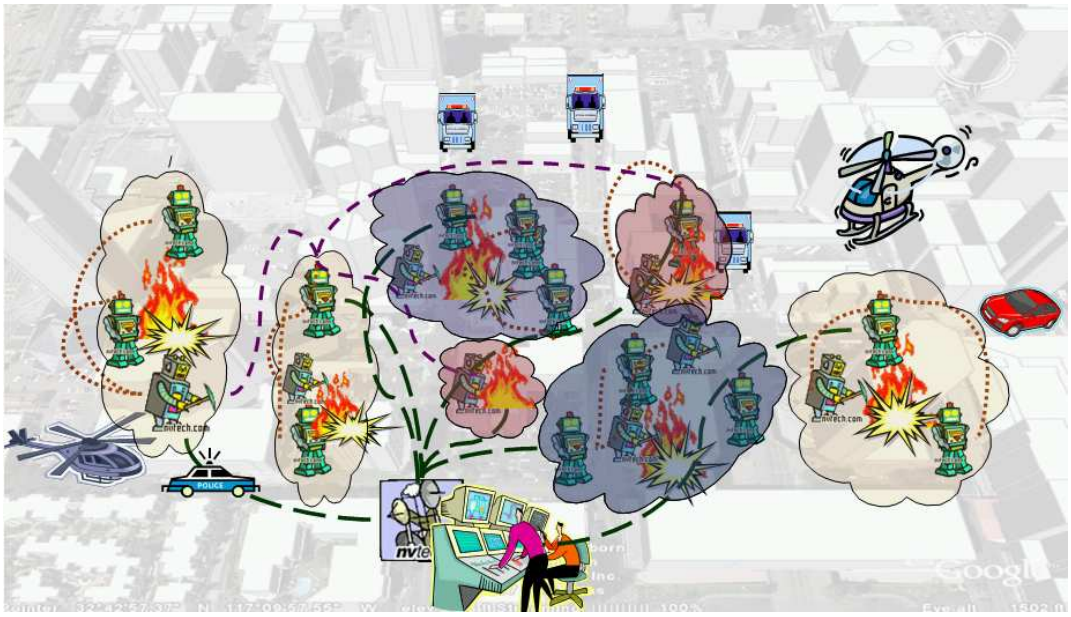


Figure 6.5 Partitioning results for robot team formation using KMeans algorithm. Since KMeans performs the partitioning on the basis of a single objective of cluster compaction, the teams identified using this algorithm are such that some of the teams are very large in size, whereas some of the teams are very small in size.

in nature) to be optimized. Some of the partitioning criteria in this domain include compaction, equi-partitioning of robots on the basis of capabilities per partition, number of units per partition, availability of average battery power per partition, or equi-distribution of workload per partition, etc. Hence, a technique for simultaneous optimization of conflicting objectives needs to be developed. In this work, we have developed a novel methodology that performs simultaneous multi-objective partitioning of robots into teams. The methodology consists of three important components:

- An iterative hill climbing partitioning algorithm
- A multi-step normal form game theoretic model
- A Nash equilibrium (NE) based solution methodology

6.2 Why Microeconomics for Robot Team Formation?

In the context of rescue robots, due to the power and communication constraints, compactness and uniform power distribution have been considered as the objectives to be optimized. Since these two objectives are conflicting, and thus convex in nature, the system can be naturally modeled as a game. Also, as shown in [13], if the payoff function is convex, a Nash equilibrium solution always exists and tends to evaluate globally optimal solutions.

In multi-emergency environments, it is desirable to ensure that all emergencies receive resources in a fair manner. Fairness has several connotations, but in this case it corresponds to a situation where each emergency receives its fair share of rescue robots. Game theory exhibits a unique property social equity or social fairness [13], which ensures that each player in the game is satisfied and the overall goals are reached. The game theoretic social equilibrium inherently ensures the optimum values of each objective with respect to other objectives, which is desirable in these scenarios.

6.3 Background

The real time applicability of mobile robots for urban search and rescue (USAR) was first recognized during the World Trade Center disaster [113]. The research in the domain of USAR, and human-robot integration (HRI) [114] has identified explicit communication among the robots as a big obstacle. This is attributed to the limited network bandwidth, limited battery power of robots, and

noisy communication channels. An effective solution to this problem is to cluster the robots into teams to ensure robustness and reliability. In [115], the authors have discussed the performance of several rescue robots at the RoboCup Rescue Real Robot League competitions, and have identified that the multiple robot cooperation, and teams of robots could maximize the search regions, and utilize and enhance the abilities of robots in search and detection missions. However, much research concentrates on the identification and optimization of task distribution, and cooperation among the robots within a team [111].

In [112], the use of stochastic game theory to model cooperation among the robot team on the basis of observation history has been demonstrated. Similarly, in [116], the authors propose a hybrid robotic communication mechanism that uses robot vision and radio signals for improved communication. In [117] the multi-robot exploration problem has been addressed from a different perspective by suggesting a KMeans based clustering of the unknown search space and allocating the space to the robots for exploration.

The problem of object partitioning has been investigated in the context of a wide range of applications, and reported in literature. Detailed surveys of these works can be found in [11, 69]. An elaborate discussion of these methods on the basis of partitioning criteria like compaction, equi-partitioning, connectedness, and spatial separation can be found in [12, 78]. The KMeans [74] is the simplest and most widely used mathematical algorithm for partitioning on the basis of compaction. It is used for creating initial partitions in our approach discussed later. Additionally, some heuristics based techniques [83, 88] and hybrid approaches [85] have been proposed in literature. However, all of these methodologies are limited to single objective optimization. In the realm of multi-objective optimization, the proposed models primarily consist of ensemble methods [89] that perform single objective optimization using different methods for different objectives, and integrate the results *a posteriori*. These methods do not exploit the real strength of simultaneous multi-objective optimization. *Microeconomic optimization* methods are capable of naturally modeling the situations of conflict and cooperation in a game theoretic setting as discussed in the previous section. It models optimization problems in a framework consisting of players with conflicting objectives competing to optimize their individual as well as the system wide utilities [27, 28]. The game is solved using Nash equilibrium based methodology that identifies a socially fair solution [36].

In this work, we identify the robot team formation problem as a multi-step normal form non-cooperative game. A subset of initial partitions identified by KMeans algorithm are modeled as players, and the remaining as resources, different combinations of robot requests by players from different resource centers as strategies, and a function of competing objectives compactness and uniform power distribution as the payoff. The partitions are updated iteratively on the basis of NE solutions until the stopping criterion is satisfied.

6.4 Microeconomic Modeling

In this section, we describe the partitioning algorithm for multi-robot team formation. Since this is an application of the multi-objective clustering approach being presented in Chapter 5, the methodology follows the same steps for most part. In this section, we will briefly discuss the steps involved in the algorithm. Please refer to Section 5.3 for detailed description of these steps.

The algorithm identifies the initial partitions using the KMeans clustering method, and if the initial partitions are not optimal, a game is formulated with the partitions as players and resources. A Nash equilibrium solution of the game identifies the optimal reallocation of robots to the partitions. The notations and terminology being used in the rest of the paper are given in Table 6.1.

6.4.1 KMeans Partitioning

This methodology requires the KMeans algorithm to identify initial as well as the updated teams of robots. The KMeans algorithm partitions the total number of robots N into partitions (K) depending upon the number of emergency locations in a region. Each team would perform the search and rescue operations at the corresponding emergency location. The steps involved in the mathematical partitioning process are:

- Initialize the random partition heads at the coordinate locations near the emergency locations.
- Calculate the distance (Euclidean in this case) of each robot from each of the partition heads, given as:

$$E^{(k)}(W) = \sum_{i=1}^{N} w_{ik} \sum_{j=1}^{d} (x_{ij} - c_{kj})^2 \tag{6.1}$$

Table 6.1 Notations for robot partitioning. The notations are used in developing the algorithm for robot teams formation on the basis of compaction for low power dissipation in communication, and equi-partitioning for uniform power distribution.

N	Total number of robots in the system
d	Total number of attributes of a robot (coordinates)
K	Total number of partitions
E_{nk}	Euclidean distance between n and k, where $n \in N$ and $k \in K$
E	Sum of the squared Euclidean distance
l_k	Number of robots in partition $k, \forall k \in K$
l_{ideal}	Number of robots per partition in a uniform power distribution situation; $l_{ideal} = \lvert N/K \rvert$
L	Uniform Power Distribution Measure; $L = \sum_{k=1}^{K} (l_k - l_{ideal})^2$
P	Total number of players; $P \subset K$
p_i	i^{th} player in a game;$\forall i \in P$
p_{-i}	The set of all the players in the game other than the player p_i
R	Total number of resource centers; $R \subset K$
r_j	j^{th} resource center in a game;$\forall j \in R$
r_{-j}	Set of all the resource centers not in the current game
U_i	Total number of strategies of a player p_i
S_i	Set of all the strategies of player p_i
s_u^i	u^{th} strategy of the player p_i; $s_u^i \in S_i$ and $u = 1, \cdots, U$
S	Strategy set consisting of all the strategies in the game; $S = \{S_1, S_2, \cdots, S_P\}$
S_{-i}	Set of all the strategy combinations of all the players other than p_i
s_v^{-i}	A strategy combination consisting of one strategy of all the players other than p_i; $s_v^{-i} \in S_{-i}$

$$E(W) = \sum_{k=1}^{K} \sum_{i=1}^{N} w_{ik} \sum_{j=1}^{d} (x_{ij} - c_{kj})^2 \tag{6.2}$$

Here, Equation (6.1) corresponds to the distance measure for the kth partition and Equation (6.2) corresponds to the total intra-partition variation.

- Assign the robot to the partition according to the sum of squared Euclidean distance (SSE) measure, as given by Equation (6.3).

$$E(W^*) = \min_{W} \{E(W)\} \tag{6.3}$$

6.4.2 Game Theoretic Partitioning of Robots

The process of identifying partitions with the objective of minimization of SSE measure adversely affects the complementary power distribution objective (denoted by L). Hence, a game is required to be formulated to simultaneously optimize all the conflicting objectives. Specifically, the process involves the identification of initial partitions using the initialization step of KMeans algorithm. These

partitions are then categorized as players and resource centers, and a game is formalized. The players in the game compete for allocation of resources (robot units) from the resource centers. The strategy of a player is modeled as a tuple consisting of the number of robots requested from every resource center. The payoff corresponding to the various strategies represents conflicting objectives. After the formulation of the game, a Nash equilibrium solution point is evaluated and the allocations are performed accordingly. After the reallocation of robots to the partitions according to the game theoretic solution, the partitions are updated using the KMeans algorithm. This complete process is repeated until optimum partitions are identified. The steps involved in the algorithm are described in Algorithm Algorithm 6.1 .

Algorithm 6.1 Microeconomic robot team clustering algorithm

Require: Locations of robots, initial number of partitions, initial allocation of robots to the partitions after KMeans

Ensure: Optimal Partitioning of robots into teams on the basis of power distribution and compaction objectives

1: **if** for each partition k **then**
2: The condition $l_k = l_{ideal}$ is satisfied, then report the solution as optimal, and exit.
Here, l_{ideal} = number of units per cluster at the uniform power distribution state
3: **else**
4: Classify the unequal partitions as players and resources:
5: Players: all k, such that $l_k \leq l_{ideal}$;
6: Resources: all k, such that $l_k \geq l_{ideal}$;
7: **end if**
8: **for** For each resource location **do**
9: Players formulate a game with their strategies, to receive units from the resource, so that the overheads are distributed among the players and the resource achieves a consistent state (l_{ideal}).
10: The game is then solved for an equilibrium solution point using Nash equilibrium algorithm
11: **end for**
12: After the reallocation, the new partition centers are identified, and the process is repeated until convergence

The generation of the strategy set involves the players trying to receive units from the resource locations on the basis of minimum cost allocation methodology, irrespective of the allocations made to the other players. However in this process, a situation may arise where location has allocated more units than its overhead. Therefore, a game is formulated and solved for that resource location, and subset of partitions that have tried receiving units from it play the game. The strategies of the players consist of the number of units they may need to lose in order to ensure that the resource location is in consistent state, i.e., it has a uniform power distribution. Due to this alternative definition of strategy,

a single step game with an exponential strategy set and large number of players is reduced to multiple games of with significantly smaller strategy sets and players. The details of the steps followed for defining the strategy set for the game theoretic formulation are given in Section 5.3.2.2.

Each strategy combination in a game has an expected utility that a player would receive. The utility is mathematically modeled as a payoff function evaluating the gain or loss a player would incur when it plays its own strategy and the other players play their corresponding strategies. The payoff function in this model captures the inter-relationship of the optimization criteria, and is modeled as a geometric mean of the total loss incurred by a player when it plays a particular strategy. Specifically, a player would have to receive units from a distant resource location if the other players request for all the units available with the current resource location. The payoff, a function of power distribution metric and compaction metric is the loss to the player when such a situation occurs. The algorithm for payoff function is given in Algorithm Algorithm 5.4 in section 5.3.2.3.

The payoff matrices evaluated during the previous step are given as input to the Nash equilibrium (NE) algorithm, which generates an output as a NE strategy set consisting of one strategy chosen for every player in the game. At the Nash equilibrium point, no player has incentive to change its strategy unilaterally. Mathematically, the NE point is given by Equation (6.4). After the equilibrium strategies are identified, the reallocation of units is performed accordingly. The game is then played for other locations in conflict and the allocations are performed accordingly. The partition medoids are then updated, and the complete process is repeated until the relative improvement in the power distributions does not supersede threshold decrease in the value of SSE.

$$po_i(s_1^*, \cdots, s_i^*, \cdots, s_{P'}^*) \geq po_i(s_1^*, \cdots, s_i, \cdots, s_{P'}^*) \tag{6.4}$$

6.5 Experimental Results

In this section, we present the experiments that were carried out to evaluate the efficacy of the methodology for robot team formation. Since there are no benchmarks available for multi-objective robot partitioning, several artificial data sets were created to simulate the real world scenarios. The performance of the microeconomic model was compared with the classical KMeans algorithm.

6.5.1 Simulation Tools and Setup

To simulate the locations of robots on a terrain the following setup was formulated:

- A two dimensional grid of size 12*12 was created, and normally distributed data sets consisting of the x-y coordinates of the robot locations on the grid were generated.
- The values of mean and variance were varied from $0 \leq \mu \leq 10$ and $\sigma = \pm 2$ respectively for each data set.
- The data sets with 35 nodes were generated, with 3 to 7 clusters per data set.
- The intra-partition similarity measures in terms of number of robots per partition were taken into consideration. For example, a data set 5_7 would have 5 partitions, each partition having the number of robots ranging from $\lfloor 0.7 * (35/5) \rfloor = 4$ to $\lfloor (0.3 * (35/5)) + (35/5) \rfloor = 9$.
- Each experimental result was an average of 200 repetitions with random gateway location initializations.
- The Nash equilibrium (NE) solution to the n-person normal form game is identified using the Simplical Subdivision algorithm, which has been identified to work consistently better than other existing NE methodologies available in literature. Based upon the simplex method, the algorithm starts with a given grid size, and converges to an approximate solution point by iterative labeling of the sub-simplexes.
- Gambit [108], an open source C library of game theory analyzer software toolkit for identification of NE solution was used as a solution methodology.

6.5.2 Analysis

Experiments were conducted to study the performance of this method in simultaneously optimizing the objective functions; the compaction measure (*SSE*) and the uniform power distribution measure (*L*). As shown in Figure 6.6, for a data set 6_7 that consists of 6 gateways, and 35 robots distributed among the gateways, our methodology identifies the partitions with the *SSE* of 58.86, and the *L* being 10.83, which contributes toward an improvement of 90.3% and 90.2% respectively from the initial values. However, for the KMeans algorithm, although the improvement in compaction is 1.3% higher

than game theoretic method, the power distribution objective is 45.1% worse. Overall, the game theory based multi-metric optimization method outperforms the KMeans algorithm in terms of simultaneous optimization of the multiple objectives.

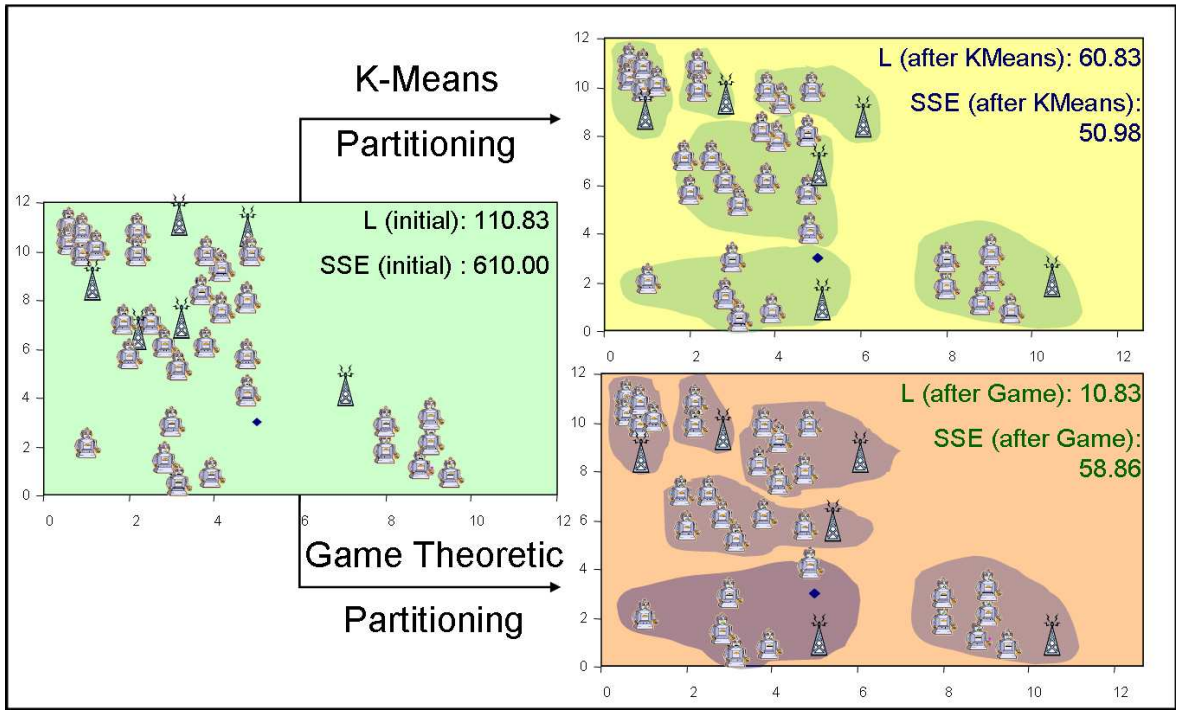


Figure 6.6 Identification of optimum sizes of the clusters and the locations of the cluster centers using game theoretic algorithm, and KMeans algorithm. Total number of robots = 35, total number of gateways = 6, and name of data set = 6_7.txt.

The average performance of the new method was also compared with the KMeans algorithm. All 35 data sets were executed and average of the outputs for improvements in *SSE* and *L* were plotted on a graph as shown in Figure 6.7. As shown, the improvement in the compaction objective is higher for the KMeans algorithm. This is intuitive since KMeans performs the partitioning only on the basis of optimization of compaction objective. However, this adversely affects the uniform power distribution objective, and is evident from the graph.

In contrast, our algorithm simultaneously optimizes both the objectives. It is important to note that both the objectives are optimized with an average improvement of more than 50% in terms of results. An interesting observation is that as the number of partitions increase, the performance of this method improves and after certain limit it degrades. This is due to the increasing dimensionality of the problem. If the number of partitions are too few, the initial partitions identified by the initial iteration

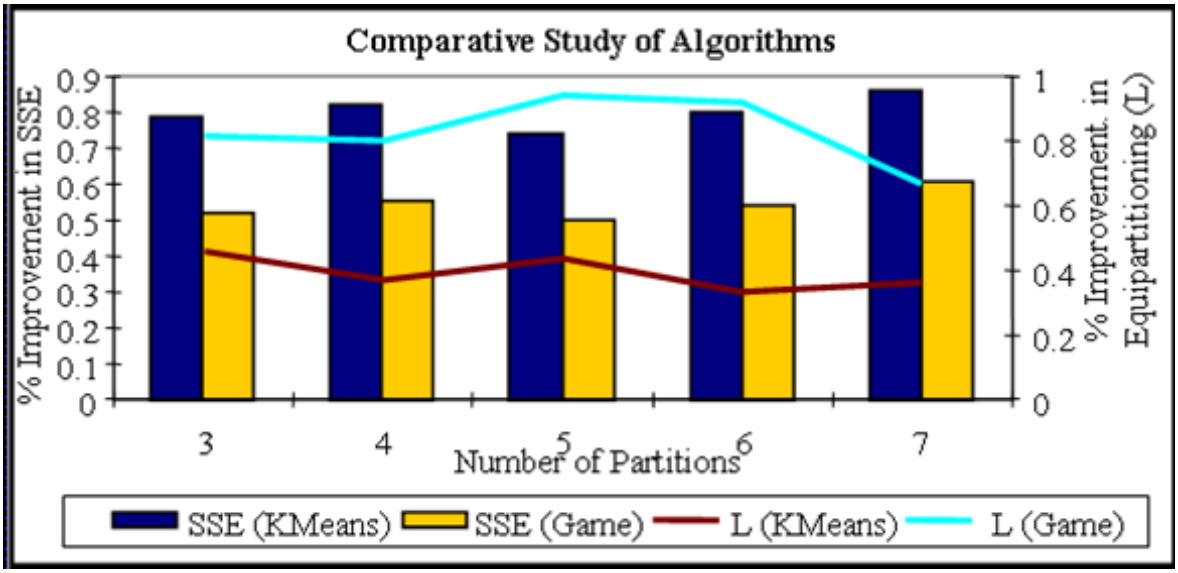


Figure 6.7 Average performance of algorithms on artificial data sets. The KMeans and the game theoretic algorithm are compared for their performance on the cluster compaction (*SSE*) and uniform power distribution (*L*) metrics.

of KMeans optimally partitions the data, and a game is not formulated often. As the partitions increase, the KMeans initialization is unable to identify optimum clusters resulting in multiple iterations of game formulation and hence simultaneous optimization of objectives. However, as the partitions increase beyond a certain limit, the number of strategies per game increase and the game theoretic model in its current form prunes the strategy set to control the dimensionality of the problem. Due to this, occasionally the strategies that are not locally optimal but have a global effect may get pruned thereby affecting the performance.

The response time of a microeconomic model largely determines its practicability in an application domain. The parameters that largely govern the response time for game theoretic model in this context include the number of players, the number of partitions, and the total number of strategies of players. Table 6.2 shows the average values of these parameters for different number of gateways. For smaller number of partitions, the initial KMeans clustering is often optimal and a game is not required to be played, and hence average number of iterations of game is less than one. The results on the simulated data sets are promising because as the number of partitions increase, the number of strategies do not increase exponentially, which is a concern with most of the problems modeled in a game theoretic framework. The linear relationship between the size of strategy set and number of partitions is attributed to the novel definition of the strategy and the modeling of the game in this context of this

work. The strategy set for a player in this model does not depend on combinations of the number of resource locations that are availing the resources, but on the number of units a player may have to lose for keeping the resource in a consistent state.

Table 6.2 Parameters affecting the game theoretic model. The inter-relationship between the important attributes of a game such as the number of players, number of strategies, number of clusters, total number of game iterations, and the execution time of the algorithm is identified.

Partitions	3	4	5	6	7
Avg. Iterations of Game	0.23	0.80	1.52	1.96	2.02
Avg. Number of Players	0.22	0.60	1.39	1.60	2.13
Avg. Number of Strategies	0.32	2.08	4.71	5.27	6.77
Response Time (sec.)	0.0003	0.0627	0.1447	0.1615	0.1968

6.6 Discussion

A novel microeconomic approach for multi-objective robot team formation problem has been developed in this research. It models the problem as a hybrid approach involving Kmeans and non-cooperative multi-player normal form game with Nash equilibrium based solution. The objective functions being considered in the model are compactness, and uniform power distribution. The simulations have been conducted using normally distributed artificial data sets. The performance of this method as compared to the KMeans algorithm conforms to the claim that our model is better suited for robot aggregation than the existing partitioning methods. The average complexity of the system is non-exponential. This is the first successful attempt in the direction of robot team formation on the basis of multiple objectives. Currently, the model is simplistic, and optimizes only two objectives simultaneously. However, the practical implementation of the model may require more objectives, like improved radio communication, minimum inter-team communication, etc. to be considered. In such scenarios, the payoff modeling would need further investigation and refinement. Also, in practice, the capabilities of each robot are different, and such considerations must be reflected in the modeling. It is required to deploy robots in several real world test scenarios to efficiently and accurately evaluate the performance of the algorithm.

CHAPTER 7

CONCLUSIONS AND FUTURE DIRECTIONS

Successful packing two billion transistors on a single chip [10] gives a clear idea about the level of miniaturization, and density of the next generation VLSI circuits. This increase in the integration uncovers numerous issues that have to be addressed by the designers in order to realize high performance, low power dissipating, and reliable circuits. Some of these concerns include the impact of process variations at nanometer level, the effect of various performance metrics on each other, and the efficiency of the circuit optimization methods. It is a challenging task to address all these issues in a single model. The focus of this dissertation is to address all these concerns in the VLSI domain, and to develop a framework that is capable of solving the current as well as next generation VLSI circuit optimization problems.

The size of an optimization problem in any engineering discipline encourages the use of clustering mechanisms to partition a large problem into smaller problems, and solve them separately. However, it is difficult to adapt the knowledge and intelligence from classical clustering disciplines to solve this problem. Specifically, in situations where the clustering needs to be performed on the basis of multiple objectives that may be competitive in nature, single objective clustering algorithms can not generate good clusters. Thus, the development of a generalized clustering mechanism for such problems is imperative.

In this dissertation, we have developed multi-metric optimization frameworks to solve the VLSI-CAD circuit optimization problems and spatial pattern clustering problems, using utilitarian methods. The specific problems being solved in this dissertation are as follows:

- A post layout gate sizing algorithm for multi-metric optimization of delay, leakage power, dynamic power, and crosstalk noise in the presence of process variations [118]. The algorithm generates a deterministic equivalent of the inherently stochastic optimization problem, while ensuring high utility levels. It is independent of the process variation distributions and can in-

corporate the impact of variations due to gate sizes as well as interconnects. The algorithm is also capable of incorporating randomness in the objective functions.

- Development of a unified mathematical programming based framework for multi-metric optimization of delay, leakage power, dynamic power, and crosstalk noise in the presence of process variation. The framework can be implemented using any mathematical programming technique, and is completely reconfigurable in terms of prioritizing or selecting the metrics to be optimized.

- Development of a simple yet effective cross-talk noise model and identification of relationships between the different performance metrics in terms of gate sizes.

- Development of a novel game theoretic clustering approach for simultaneous multi-metric clustering of spatial data objects. A general framework is developed that can incorporate any number of conflicting clustering objectives.

- The game theoretic clustering approach is applied to solve the multi-objective robot team partitioning problem in multi-emergency search and rescue missions [119]. The partitioning is performed on the basis of cluster compaction and uniform power distribution.

The utilitarian methods being applied in this dissertation possess certain unique attributes that have made their application suitable to solve these problems, and the identification of these methods is an important contribution of this dissertation. The expected utility based approaches change the perspective of solving the stochastic gate sizing problem with random constraints to a deterministic risk minimization problem with an objective of maximization of expected utility of the satisfaction of the constraints. This transformation significantly reduces the time complexity of the algorithm, while maintaining a high yield. This is a prime contribution of this dissertation. The modeling of a clustering problem in a game theoretic framework is novel. The new definition of strategies for the players has contributed toward significant reduction in the time complexity of the algorithm. A novel definition of payoffs as a function of equi-partitioning and compaction is unique. The application of spatial clustering algorithm for the robot team formation is a practical problem, and this is one of the very few works that have addressed the problem with this perspective.

The approaches presented in this dissertation are novel and have wide applicability in the various areas of research. Some of the future directions to improve over this dissertation work, and other interesting research ideas are listed as follows.

- The multi-metric optimization model for VLSI circuit optimization presented in this dissertation incorporates four metrics that have been optimized. Additional metrics like security and reliability etc. can be incorporated easily once the relationship between the metrics in terms of gate sizes is identified.

- The expected utility based methods can be utilized for various circuit optimization techniques like buffer insertion or repeater insertion and wire sizing. These methods also find applications in solving the multi-metric optimization problems using gate sizing at the logic level or RTL level.

- The expected utility based optimization presented in this dissertation assumes that scarce information in terms of only mean and variance of the process variations is available. However, if more information in terms of coefficient of correlations is also available, the model can be further extended to incorporate such information and formulate a linear programming equivalent model with quadratic constraints [94].

- The VLSI multi-metric optimization problem contains an objective function that is deterministic in nature. However, the expected utility based method is capable of solving the problems with random objectives also. This is an interesting future work for multi-metric optimization with different levels of randomness in the individual metrics. Such a solution will give a frontier of solution points.

- The game theoretic spatial clustering algorithm in its current form is capable of clustering medium sized data sets. This is attributed to the non-linear increase in the number of strategies as the number of players increase. However, if better techniques are incorporated to aggressively prune the strategy set, the algorithm would be able to cluster larger data sets.

- An alternative notion of applying the game theoretic method to the clustering problem is to consider the objectives as the players. It would be interesting to see the changes in the optimization

performance, since the number of players in that scenario would be constant, but the strategy set may be larger.

- The game theoretic clustering approach has several useful applications. One such application is in the domain of ad-hoc and sensor networks. The ad-hoc networks need clusters to be formed with the objective of minimizing inter- as well as intra-cluster communication. To satisfy these requirements, each cluster designates one of the nodes as a gateway for inter-cluster communication and one node as a cluster head for intra-cluster communication. Game theoretic clustering approach can be utilized to solve this clustering problem.

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